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PREDICTION ERROR ON THE SYSTEMATIC RISK OF A SECURITY AND THE VALUE OF ACCOUNTING INFORMATION TO THE INDIVIDUAL INVESTOR

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I hereby recommend that this dissertation prepared under my direction by Don Ray Hansen

entitled Prediction Error on the Systematic Risk of a Security and the Value of Accounting Information to the Individual Investor

be accepted as fulfilling the dissertation requirement for the degree of Doctor of Philosophy

Don Vickrey
Dissertation Director

8/10/77
Date

As members of the Final Examination Committee, we certify that we have read this dissertation and agree that it may be presented for final defense.

R. W. Fastett	August 9, 1977
Wilbur B. Barrett	August 10, 1977
Dwight D. Summers	August 10, 1977

Final approval and acceptance of this dissertation is contingent on the candidate's adequate performance and defense thereof at the final oral examination.

STATEMENT BY AUTHOR

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SIGNED: Roy R. Hansen

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ABSTRACT

The objective of the study is to determine whether or not prediction errors on beta can be eliminated at the portfolio level via diversification and, consequently, determine if accounting information has potential value to the individual investor in estimating the systematic risks of firms. The study examines the problem analytically and empirically. First, it was determined analytically whether prediction error could be eliminated by diversification under two assumptions about the probability distributions of the beta predictors: (1) the symmetric-stable-Paretian, identical characteristic exponent, stable-beta assumption, and (2) the contaminated normal, nonstationary-beta assumption.

The study shows that portfolio-level prediction error can (cannot) be diversified away from the first (second) assumption. The conclusion that prediction error can be eliminated under the first assumption implies ultimately that there is no incentive for the individual investor who holds a well diversified portfolio to use accounting information in predicting beta if this assumption is valid. The conclusion that prediction error cannot be diversified away under the second assumption implies that such information may be useful to the individual in predicting beta if this assumption is valid.

In order to determine if the effect of prediction error at the portfolio level is of any practical significance, an empirical investigation of the prediction error problem is also conducted. Two sets of portfolios are studied. For one set of portfolios, securities are

identified that have indications of stable betas for two consecutive time periods. The other set of portfolios has a substantial proportion of securities with unstable betas for the first time period and stable betas for the second time period. The betas of the first time period are used to predict the betas of the second time period.

Using four prediction error measures, the impact of prediction error at the portfolio level is evaluated. For both sets of portfolios, the evidence indicates that diversification is unable to eliminate prediction error. Thus, it appears that even individuals with well diversified portfolios might use accounting information to help improve their estimates of the portfolio beta.

CHAPTER 1

INTRODUCTION

In recent years researchers have expended a great amount of effort on the efficient market hypothesis. This hypothesis states that equilibrium prices of securities fully reflect some information set at any point in time. Implicit in this statement of the efficient market hypothesis are the notions that the market reacts instantaneously in an unbiased manner to new information and, therefore, the conclusion that the market contains no undervalued or overvalued securities.

Fama (1970) refines the efficient market hypothesis by defining market efficiency relative to three levels of the information set; the weak, the semi-strong, and the strong forms. The weak form states that equilibrium prices fully reflect the sequence of historical prices. The semi-strong form states that equilibrium prices fully reflect all publicly available information. The strong form adds inside information, and, thus, in the strong form, equilibrium prices fully reflect all available information.

External accounting information is a subset of the semi-strong information set; therefore, the semi-strong form is of particular interest to accountants. Given that a considerable body of empirical research has indicated that the market is efficient in the semi-strong form, an evaluation of the role of accounting in such a market is justified.

Accounting Information, the Individual Investor, and Efficient Capital Markets

According to Beaver (1972, pp. 416-420) accounting information in an efficient capital market has two possible roles. First, it is possible that accounting information can be used to assist an individual investor in security analysis so that he may select an optimal portfolio. And, secondly, accounting information may be used to assist in establishing a set of equilibrium prices so that resources are allocated optimally among firms and securities are allocated optimally among investors. This study focuses only on the first role, the value of accounting information to the individual investor.

Since there are no undervalued or overvalued securities in a semi-strong efficient market, accounting information cannot be used to identify such securities. Thus, if accounting information is to be of value to an individual investor in making his portfolio decision, its worth must necessarily be related to some other factors. Capital market theory and the related capital asset pricing model (Fama and Miller 1972; Sharpe 1970) provide a framework from which it is possible to draw implications concerning the role of accounting information in an efficient market. Some of the essential features of capital market theory will now be reviewed; it is felt that this review is a necessary prerequisite to the presentation of the objective of this study.

The Framework of Capital Market Theory

Capital market theory assumes that investors all are averse to risk and maximize expected utility. That is, for a two-period[1] consumption-investment decision, each investor behaves as if he were trying to maximize expected utility with respect to a utility of consumption function, $U(c_1,c_2)$, where c_1 and c_2 represent the consumption of periods one and two, respectively. The consumption of period two is determined by the investment at the beginning of period one. The investment is assumed to be in securities of firms whose market value at the beginning of period two determine period two's consumption.

Moreover, under uncertainty, each investor is faced with a set of probability distributions on market values of firms at the beginning of period two (which are not known, but must be estimated). These distributions are the basic objects that must be priced and cleared from the capital market at the beginning of period one. It is further assumed that no decisions are made until an equilibrium set of prices has been determined. And, of course, markets for consumption goods and investment are assumed to be perfect, which, according to Fama and Miller (1972, p. 277) embodies the following:

> . . . all goods and assets are infinitely divisible; any information is costless and available to everybody; there are no transaction costs or taxes; all individuals pay the same price for any given commodity or asset; and no firm is large enough to affect the opportunity set facing consumers. In short, . . . individual consumers and firms are assumed to be price takers in frictionless markets.

The investment decision for an individual consists of selecting the best probability distribution on market values of firms. That is, the one which maximizes the expected utility of the individual. In principle, this decision consists of examining, in complete detail,

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1. Fama and Miller (1972, p. 323) observe that given basically the same assumptions of a two-period model, a consumer's behavior is essentially the same for a multiperiod model.

each probability distribution associated with each possible consumption-investment choice. This process, however, has no practical meaning as there are millions, and, in fact, countless numbers of alternative distributions (Sharpe 1970, p. 23). If comparisons of probability distributions are to be made, only the essential characteristics can be considered.

Typically, two numbers are used to characterize the probability distributions. It is assumed that individuals can summarize their investment opportunities in terms of means and some measure of dispersion, usually the standard deviations of the return distributions.[2] On the basis of only these two parameters of the return distribution, the assumption is that an individual can rank a portfolio relative to other portfolios.

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2. Conceptually, it matters little whether one speaks of return distributions or of distributions of market values. Once one of the two is known, so is the other. To see this relationship, let

w_1 = total wealth at the beginning of period one,
w_2 = total wealth at the beginning of period two,
w_1-c_1 = investment at the beginning of period one, and
R_2 = the one-period return at the beginning of period two per dollar of investment at the beginning of period one.

At the beginning of period one, an investor allocates a portion of his total wealth, w_1, to investment in a portfolio whose market value determines the investor's period two wealth, w_2 (Fama and Miller 1972, p. 149). Under uncertainty, w_2 is considered to be a random variable (i.e., there exists a probability distribution on the market values of firms). Or if it is desired, at the beginning of period two, the individual's wealth level can be represented by:

$$w_2 = (w_1 - c_1)(1 + R_2),$$

where R_2 is also viewed as a random variable (i.e., there exists a probability distribution of returns).

If this ranking is to be meaningful, then the two parameters should fully describe the return distributions. Also, if valid comparisons are to be made, the portfolio return distributions must be of the same two-parameter type. Since component assets are themselves portfolios, the preceding statement is equivalent to assuming that the stability property holds. That is, the sum of random variables, where each random variable has the same distribution form except for origin and scale, also has this same distribution form except for origin and scale.

One class of distributions which satisfy these requirements is the family of symmetric stable Paretian distributions[3] (Fama and Miller 1972, p. 261). A symmetric stable Paretian distribution has three parameters, c, m, and s,

where:

c = the characteristic exponent,

m = a measure of central location, and

s = a measure of dispersion.

The characteristic exponent identifies the type of stable distribution. For example, when $c = 2$, the stable distribution is the normal distribution. Note that the normal distribution is fully described by its mean and standard deviation. Also, the weighted sum of normal random variables is again a normal random variable. Interestingly, early empirical evidence (Fama 1965a, pp. 34-105, Mandelbrot and Taylor 1967, pp. 1057-1062) indicates that return distributions appear to be members of the symmetric stable Paretian family.

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3. This family of distributions is discussed more completely in Chapter 3 of this study.

In the two-parameter portfolio theory model, the dispersion measure is regarded as the measure of the riskiness of the investment. The measure of central location is viewed as the measure of the expected return of the investment. Hence, investment analysis reduces to the assessment of the risk-return characteristics of return distributions (i.e., portfolios) that exist at the beginning of period two. In order to price and clear these return distributions at the beginning of period one, the expected return and risk of each of the distributions must be assessed. The ex ante values of both parameters are necessary in order for the market to clear.

Given that investors act on the basis of predictions stated in terms of the two parameters, expected return and risk, an interesting problem to consider is the equilibrium relationship between risk and expected return for assets. A model, describing how specific asset prices are established was developed by Sharpe (1964), Lintner (1965), and Mossin (1966), and is commonly referred to as the capital asset pricing model. A brief description of the model is given below. This description completes the review of some of the more important features (i.e., relative to the purposes of this study) of capital market theory. Following the description, the role of accounting information within the framework of capital market theory is examined.

The Capital Asset Pricing Model

The capital asset pricing model embodies all of the assumptions in the two-parameter portfolio theory model, and is usually restricted by two additional assumptions. First, it is assumed that investors possess homogeneous expectations; that is, all investors use the same set of predictions. Secondly, it is assumed that there is a riskless asset (i.e., one with no dispersion) and that an investor can borrow or lend as much or as little as he wishes at the riskless rate of interest. According to the model, the only variable that determines the differential riskiness among securities is the systematic risk. Systematic risk is a measure of a security's contribution to the overall risk of a portfolio and is almost always referred to by the symbol beta.[4] The model, in equation form is:

$$E(R_i) = R_f + [E(R_m) - R_f]\ \beta_i \qquad (1.1)$$

where:

$E(R_i)$ = the expected return of asset i,

R_f = the return on a riskless asset,

$E(R_m)$ = the expected return of the market, and

β_i = the systematic risk of security i.

Thus, the model yields a picture of market equilibrium that implies a measure of risk for individual assets and a relationship between risk and equilibrium expected return.

Implications of Capital Market Theory for Accounting

According to the two-parameter portfolio theory model of capital market theory, investors need to know, in an ex ante sense, the probability distributions of beginning of period two returns. That is, in order to clear, the market needs to know the means and variances (or

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4. A more rigorous treatment of systematic risk is given in Chapter 2.

some other measure of dispersion) of all return distributions. It is assumed by capital market theory that a given set of production decisions determine the set of return distributions at the beginning of period two.

The role of accounting information can be related to the above concepts. Accounting information might be used by investors to help them assess the risk-return characteristics of the set of period two return distributions, and, thus, be of use in helping to set equilibrium prices. And, within the framework of this perfect world described by capital market theory, investors simultaneously select their optimal portfolios. That is, investors are able to estimate the risk-return characteristics of all portfolios, and then select the one which maximizes expected utility. Thus, the two possible roles of accounting information mentioned earlier in this section become somewhat indistinguishable. Ultimately, the view could be taken that optimal portfolio formation is a byproduct of the process of establishing equilibrium prices, and that accounting information has potential value only at the social level.

At any rate, investors are not able to assess the parameters of the future return distributions without error. Thus, as inferred above, accounting information may be useful in improving the predictions of the risk-return parameters of future return distributions. That is, accounting information may help an individual investor in selecting his optimal portfolio (this is true whether one is talking of the societal level or the individual level, which as pointed out may effectively be the same with respect to the role of accounting information).

Beaver (1972, pp. 420-423) discusses the implications of capital market theory relative to the value of accounting information to an individual investor. Implicit in the discussion is a transition from the perfect world of capital market theory to the world of reality. In effect, he draws conclusions concerning the real world from the framework of capital market theory. First, he observes that the relevant level of concern for an investor is the portfolio level rather than the individual security level.[5] That is, security specific information is relevant only to the degree to which it impacts on the portfolio parameters, risk and return. Further, within the context of the capital asset pricing model, the only variable that determines the differential riskiness among securities is the systematic risk, beta.

Within this same context, estimating the equilibrium expected return of a security requires the assessment of the return on the riskless asset, R_f, the expected return on the market portfolio, $E(R_m)$, and the systematic risk, β_i. However, since the economy wide variables, R_f and $E(R_m)$, are common to the valuation equations of all securities, security analysis reduces to the prediction of the value of the systematic risk. More specifically, he claims that the role of accounting data is its predictive ability with respect to beta. In explicit terms, Beaver (1972, p. 423) states:

> Moreover, in an efficient market, the only potential value of accounting information to the individual investor would be the assessment of the risk (and hence, expected return) associated

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5. Portfolio formation can reduce dispersion of the return distribution. It can virtually eliminate the risk peculiar to individual securities. For risk-averse investors, this process of dispersion reduction (called diversification) is obviously desirable.

with a given portfolio, which in turn would involve estimation of the systematic risk component for the individual securities that constitute the portfolio.

Beaver's conclusions appear to be overstated. Expectations on both expected return and risk are needed before the market can clear. The capital asset pricing model describes the after the fact equilibrium position of the market. Thus, a safer, and perhaps more sensible conclusion is that accounting information might be used in improving predictions with respect to both risk and expected return.

Although Beaver's conclusions may have been too strong, it is certainly true that one possible role of accounting information is its predictive ability with respect to beta. Several researchers have recognized this potential role of accounting information and have investigated its ability to reduce prediction error on beta. Some of these studies are discussed below.

Prediction Error on Beta and Accounting Information

The study of Beaver, Kettler and Scholes (1970) found that accounting data contain information on systematic risk. Specifically, they found significant association between market determined risk measures and accounting estimates of risk. They also used accounting variables as instrumental variables in the prediction of future market betas on the basis of past estimated betas and showed that this approach yielded better predictions (i.e., reduced prediction error) than the direct use of past betas. They assumed, perhaps unwarrantedly, that the betas were stable.

Rosenberg and McKibben (1973) used both historical returns and historical accounting variables to predict the distribution of future returns. They did not assume that $\beta_{it} = \beta_i$ for all t. Instead they thought that beta would vary in response to changes in the characteristics of the firm (accounting variables) and in the market's perception of the firm. The assumption was made that beta was related linearly to a set of descriptors representing the aforementioned characteristics. This linear expression was substituted into the market model for beta, and ordinary least squares (OLS) was used to determine the coefficients of the various descriptors. A total of 32 descriptors was used. Again, a strong relationship between systematic risk and accounting information was discovered. Moreover, it was again discovered that prediction error, as measured by the mean square error in forecasting returns, was smallest for the prediction model using accounting variables. Other studies which have found a significant association between beta and accounting variables are Ball and Brown (1969) and Beaver and Manegold (1975).

Prediction Error and Diversification

The studies just cited imply that accounting data may be of value to the individual investor in forming his portfolio, since it allows a better assessment of the desired risk-return characteristics. However, Beaver (1972, pp. 422-426) also suggests that accounting data may be of no value to the individual investor, if prediction errors on beta can be diversified away in the same manner as individualistic risk. Downes and Dyckman (1973, p. 312), after noting that risk information conveyed by accounting numbers is one possible area of importance for accounting in an efficient market, suggest:

> To the extent individuals are (or can become) sufficiently diversified that such risk predictions are of little value, the function of accounting takes on more of its usefulness from a societal viewpoint and needs to be evaluated on its cost of providing information to the entire market relative to alternative information systems.

Also, with respect to this issue, the Report of the Committee on Accounting Theory and Verification (1971, p. 76) asserted that prediction errors at the individual level are uncorrelated and can be diversified away.

Thus, all of the above authors at least imply that it is possible that prediction errors on beta may tend to "wash out" at the portfolio level (as the number of securities increase). Intuitively, this condition requires a canceling effect such that, in the aggregate, the net effect of prediction error essentially vanishes. For a well-diversified portfolio, probably it would be senseless to use accounting data to reduce prediction error because of the related cost.[6] In other words, such an investor would not need to use accounting information to help him assess the risk of his portfolio. Thus, it is conceivable that the value of accounting information, relative to risk, exists only at the societal level.[7] That is, the value of accounting information would be viewed within the entire set of investors, and, in fact, would incorporate the entire society.

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6. Risk predictions not involving accounting information may be more costly than those using accounting information.

7. Henceforth, accounting information is defined as the risk information that accounting data contain.

The evaluation of accounting within this context (i.e., at the societal level) is clearly different from the traditional individual investor oriented approach usually assumed.[8] Resources currently being spent on determining new and better ways of presenting accounting information probably ought to be used to evaluate the role accounting has in setting equilibrium prices. Thus, the ability to diversify out of prediction error has important implications for accounting. The claim made by several authors (cited earlier) that it is possible to diversify enough to eliminate prediction error, ought to be carefully investigated. The objective of this study is to investigate the effect diversification has on prediction error, and, consequently, determine if accounting information has potential value to an individual investor in estimating

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8. The American Institute of Certified Public Accountants published a report (1973, pp. 61-66) in which the basic objectives of financial statements were listed. Fundamentally, the basic objectives (as summarized in Welsch, Zlatkovich and White (1976, p. 3) are:

(1) To provide information useful for making economic decisions.

(2) To serve primarily those users who have limited authority, ability, or resources to obtain information and who rely on financial statements as their principal source of information about an enterprise's economic activities.

(3) To provide information useful to investors and creditors for predicting, comparing, and evaluating potential cash flows in terms of amount, timing, and related uncertainty.

(4) To provide users with information for predicting, comparing, and evaluating enterprise earning power.

(5) To supply information useful in judging management's ability to use enterprise resources effectively in achieving the primary enterprise goal.

(6) To provide factual and interpretative information about transactions and other events which is useful for predicting, comparing, and evaluating enterprise earning power. Basic underlying assumptions with respect to matters subject to interpretation, evaluation, prediction, or estimation should be disclosed.

the systematic risks of firms.[9] More specifically, the principal purpose of this study is to determine whether or not prediction errors on beta can be eliminated at the portfolio level. If they cannot be eliminated, then accounting information may be of use in reducing prediction error.

Summary of Content

The study consists of seven chapters. Chapter 1, the introductory chapter, develops the statement of the objective of the study and provides a description of its content. The goal of this study is to determine whether prediction error on the systematic risk of securities can be eliminated at the portfolio level by the process of diversification.

Chapter 2 provides a review of the principal of diversification. First, diversification within the context of the original Markowitz model is discussed. The discussion of diversification is then expanded to include the market model. This chapter also gives a description of a study which demonstrates that diversification is effective in reducing prediction error in a symmetric stable Paretian market.

Chapter 3 provides a detailed analytical discussion of the prediction error problem. This chapter mathematically describes the prediction error problem, specifies the probability distributions

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9. It is acknowledged that accounting information may be of use to investors who cannot diversify. Thus, in practical terms, the problem being investigated relates only to investors who can and do diversify. However, in theory, all investors <u>should</u> diversify since the market does not compensate an investor for bearing individualistic risk.

considered, defines and justifies the measures of prediction error, identifies the prediction model, and then analytically examines the effect diversification has on prediction error. The analytical discussion illustrates the need for empirical investigation to study the effects of diversification and also provides part of the foundation for this later work.

Chapter 4 describes the design and methodology of the empirical investigation described at the end of Chapter 3. The sample of securities is first described. Then, criteria for membership in two different sets of portfolios are specified. A brief description of the statistical tests used to determine if the criteria are satisfied is given. This description is followed by a summary of the application process of these tests. Finally, the chapter discusses the formation of the various portfolios that were studied empirically.

Chapter 5 defines and discusses the four prediction error evaluation criteria that are used to determine if prediction error is eliminated by diversification (i.e., in the empirical setting). The four criteria are the statistical tests of nonstationarity, average percentage deviation, coefficient of variation, and the root prediction error. Each of the criteria receives individual treatment.

The analysis of the empirical results is presented in Chapter 6. The analysis is broken into two major parts and one minor part. Each major part is concerned with a set of portfolios described in Chapter 4. A brief comparative discussion of the results for the two sets of portfolios constitutes the minor part.

In Chapter 7, the earlier chapters are first summarized. Then the major conclusions of the study are developed. Finally, suggestions for further research are indicated.

CHAPTER 2

THE PRINCIPLE OF DIVERSIFICATION

To provide the background for examining diversification and prediction error, it is necessary to examine the formal definition of diversification. The goal of this chapter is to provide this definition. Diversification within the context of the Markowitz model is discussed first. Then, the need for simplification of the model is developed. The market model, which satisfies this need, is presented and diversification is illustrated using the market model. Also, the problem of diversification in a symmetric stable Paretian market is discussed using the market model as the frame of reference.

The Markowitz Model and Diversification

The Markowitz expected return-variance of return rule is the foundation of modern portfolio theory. The rule makes possible the explanation of the existence of diversified portfolios. The rule states that an investor should select his portfolio of assets so that he maximizes the expected value of future return for a given variance or that he minimizes the variance of future return for a given expected value of future return (Markowitz 1952). In the Markowitz model, risk is equated with variance.

The Markowitz rule implies diversification. The reason becomes evident as one examines the risk and return of a portfolio and the effect diversification has on risk. Let P_i be the proportion of the total investment that is invested in security i, where $\Sigma P_i = 1$. Usually, for purposes of simplicity, P_i is assumed to be 1/N, where N represents the number of securities constituting the portfolio.[1] Throughout this study, P_i is assumed to be 1/N. Consider a portfolio made up of 1/N invested in each of N securities with expected values of anticipated future return denoted by E_i and variances of future return denoted by V_i. The return on security i is designated by R_i. The covariance between R_i and R_j is designated by C_{ij}. For a portfolio, the expected return, E_p, and variance, V_p can be expressed as:

$$E_p = E\left(\frac{R_1}{N} + \frac{R_2}{N} + \ldots \frac{R_N}{N}\right)$$

$$= \frac{1}{N}(E_1 + E_2 + \ldots E_N)$$

$$= \sum_{i=1}^{N} \frac{E_i}{N} \tag{2.1}$$

and

$$V_p = \mathrm{Var}\left(\frac{R_1}{N} + \frac{R_2}{N} + \ldots \frac{R_N}{N}\right)$$

$$= \left(\frac{1}{N^2}\right)\Sigma V_i + \left(\frac{1}{N^2}\right) \sum_{i}\sum_{\substack{j \\ i \neq j}} C_{ij} . \tag{2.2}$$

Equation (2.2) has been decomposed into two components: $(1/N^2)\ \Sigma V_i$ and $(1/N^2)\Sigma\ \Sigma\ C_{ij}$. The effect of diversification on risk is seen by examining the behavior of these two components as N becomes

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1. The assumption that $P_i = 1/N$ is not crucial for any of the results concerning diversification. The same results follow for any P_i (Markowitz 1952, pp. 77-91).

arbitrarily large. The first component approaches zero, while the second approaches the average covariance between the future return on each of assets constituting the portfolio. The proof of this conclusion is as follows:

$$V_p = \left(\frac{1}{N} \right) \Sigma \frac{V_i}{N} + \left[\frac{N(N-1)}{N_2} \right] \sum_{i} \sum_{\substack{j \\ i \neq j}} \frac{C_{ij}}{N(N-1)}$$

$$= \left(\frac{1}{N} \right) \bar{V}_i + \left[\frac{(N-1)}{N} \right] \bar{C}_{ij} \quad .$$

Now let $N \rightarrow \infty$:

$$V_p = \lim_{N \rightarrow \infty} \left(\frac{1}{N} \right) \bar{V}_i + \lim_{N \rightarrow \infty} \left[\frac{(N-1)}{N} \right] \bar{C}_{ij}$$

$$= \bar{C}_{ij} \; .$$

From the analysis just completed, it is evident that the risk of the portfolio is the value of the average covariance of all pairs of securities in the portfolio. The dispersion of the portfolio distribution is less after diversification than before. That is,

$$(1/N)\bar{V}_i + [(N-1)/N] \; \bar{C}_{ij} \geq \bar{C}_{ij} \quad .$$

Subtracting $[(N-1)/N] \; \bar{C}_{ij}$ from both sides of the inequality and multiplying both sides by N yields:

$$\bar{V}_i \geq \bar{C}_{ij}$$

or

$$\Sigma V_i/N \geq \left(\sum_{i} \sum_{\substack{j \\ i \neq j}} r_{ij} \; V_i^{1/2} \; V_j^{1/2} \right) \; / \; [N(N-1)]$$

where r_{ij} is the correlation between security i and security j. Next, multiplying both sides by N(N-1) gives the following result:

$$(N-1) \Sigma V_i \geq \sum_i \sum_{j, i \neq j} r_{ij} V_i^{1/2} V_j^{1/2} .$$

Now if $(N-1) \Sigma V_i \geq \sum_i \sum_{j, i \neq j} V_i^{1/2} V_j^{1/2}$ then the above inequality is true since $\sum_i \sum_{j, i \neq j} V_i^{1/2} V_j^{1/2} \geq \sum_i \sum_{j, i \neq j} r_{ij} V_i^{1/2} V_j^{1/2}$. And this inequality, i.e., $(N-1) \Sigma V_i \geq \sum_i \sum_{j, i \neq j} V_i^{1/2} V_j^{1/2}$, is easily shown to be true since

$$(N-1) \Sigma V_i - \sum_i \sum_{j, i \neq j} V_i^{1/2} V_j^{1/2} \geq 0$$

and this expression converts to

$$\sum_i \sum_{j, i \neq j} (V_i^{1/2} - V_j^{1/2})^2 \geq 0$$

which is obviously true.

A meaningful measure of the contribution of an individual security to the overall risk is the average covariance with the security in the portfolio less the average covariance without the security in the portfolio.

Efficient Portfolios

Portfolios which satisfy the Markowitz rule are called efficient portfolios. The task of portfolio analysis is to determine the set of efficient portfolios. Sharpe (1970, pp. 45-73) provides a detailed description of the procedures necessary to determine this set. An abbreviated description will be given here.

Any security or portfolio can be represented by a point in E_p, V_p space. Depending on the constraints placed on an investor only certain portfolios will be feasible. Points representing feasible portfolios will fill some region in the E_p, V_p space. Figure 2.1(a) depicts a possible region. The region is convex[2] along its upper border. Considering the Markowitz rule and the geometry of the region, one notes that the efficient set lies along the border CD. This border is called the efficient frontier.

Assume than an investor's indifference curves are linear and parallel as shown in Figure 2.1(b), where, as customary, the higher the line the more desirable the situations lying along it. The equation of any one of the indifference curves is $V_p = g + hE_p$, where h indicates the slope of the line and g the horizontal intercept. The best indifference curve is the one farthest to the left as indicated by line four in Figure 2.1(a). The objective of the investor is to minimize g which would then identify line four. Upon rewriting the equation for the indifference curve, yielding $g = V_p - hE_p$, the objective becomes:

$$\text{Minimize } g = -hE_p + V_p \text{ .}$$

---

2. The shape of the region itself, is, of course, determined by the constraints each investor faces. The convexity of the upper border results from considerations of the requirements of efficient portfolios (Sharpe 1970, pp. 52-53).

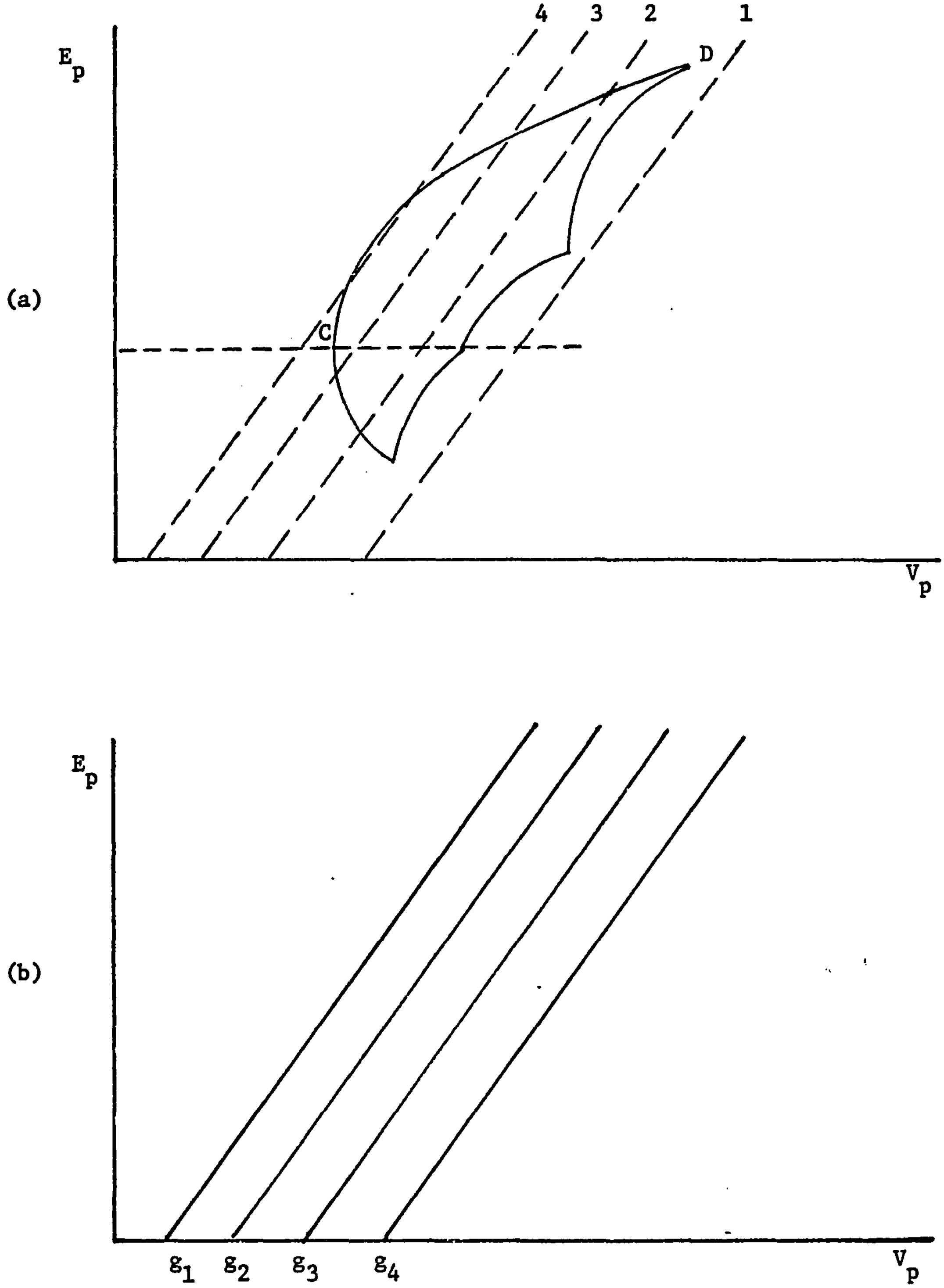

Figure 2.1. Indifference Curves and the Efficient Frontier.

One obvious constraint to the above optimization problem is that $\Sigma P_i = 1$. Other constraints are also possible. In order to solve this quadratic programming problem a total of $(N^2 + 3N)/2$ inputs are needed:

Expected Returns	N
Variances	N
Unique Covariances	$(N^2 - N)/2$
Total	$(N^2 + 3N)/2$.

The large number of inputs required by the Markowitz model restricts the use of the model because of the cost of gathering and processing the information required. Sharpe (1963) suggested a simplification which made the Markowitz portfolio model more usable. The simplified model for portfolio analysis that he developed is now considered.

The Market Model

Since almost all securities are significantly correlated with the market as a whole, Sharpe (1963) suggested that a satisfactory simplification would be to abandon the covariances of each security with each other security and to substitute information on the relationship of each security to the market. To accomplish this objective, a simple stochastic model relating the return of a security in time t to the average return of all securities in time t was developed. The model, which is generally called the market model, was assumed to be of the following form:

$$R_{it} = \alpha_i + \beta_i R_{mt} + e_{it} \tag{2.3}$$

where:

α_i = the intercept of the linear relationship between R_{it} and R_{mt},

β_i = the slope of the linear relationship between R_{it} and R_{mt},

R_{it} = the return of security i in period t,

R_{mt} = the return of the market in period t,

e_{it} = the residual, reflecting the imperfect linear relationship between R_{it} and R_{mt}.

In theory the market return is the aggregate return on all investment assets that can be held. However, in practice, this composite usually is approximated by a published index such as the Standard and Poor's 500. The residual is considered to be a random component with the following properties:

$E(e_{it}) = 0$: Covar $(R_{mt}, e_{it}) = 0$; and, Covar $(e_{it}, e_{jt}) = 0$ for all $i \neq j$.

Using the market model, fewer data inputs are needed to solve the investor's problem. The calculations now needed are:

Expected Market Return	1
Constants (α_i, β_i)	2N
Variances (of R_{mt} & e_{it})	N + 1
Total	3N + 2 .

The simplification represents a savings of (N-4)(N+1)/2 calculations, which for N large is a considerable improvement. For example, if N = 100, then 4848 less calculations are required.

The Market Model and Diversification

The market model can also be used to illustrate the benefits of diversification. Consider, again, a portfolio made up of N securities with an equal amount invested in each. Substitute for R_{it} in equations (2.1) and (2.2), the right hand side of equation (2.3). Upon simplification,

$$E_p = \bar{\alpha} + \bar{\beta}\, E(R_{mt}) + \frac{\Sigma e_{it}}{N} \tag{2.4}$$

and

$$V_p = \bar{\beta}^2 Var(R_{mt}) + \left(\frac{1}{N^2} \right) \Sigma\, Var(e_{it}) \tag{2.5}$$

where:

$$\bar{\alpha} = \Sigma \frac{\alpha_i}{N} ,$$

$$\bar{\beta} = \Sigma \frac{\beta_i}{N} ,$$

$E(R_{mt})$ = expected return of the market,

$Var(e_{it})$ = variance of the residual,

$Var(R_{mt})$ = variance of the market.

As N gets large, V_p of equation (2.5) approaches $\bar{\beta}^2\ Var(R_{mt})$ so that the individualistic risk, $Var(e_{it})$ is washed out. $Var(R_{mt})$ is common to all portfolios so $\bar{\beta}^2$ becomes a measure of the risk of each portfolio. Since $\bar{\beta}$ is made up of individual betas, beta becomes a measure of security i's contribution to the overall portfolio risk. Since $\bar{\beta}^2\ Var(R_{mt})$ is clearly less than $\bar{\beta}^2\ Var(R_{mt}) + (1/N^2)\ \Sigma\ Var(e_{it})$, diversification is again shown to be a dispersion reducing activity.

Stable Paretian Market

The assumption, implicit in the previous analysis, is that the expected returns and variances all exist. However, with one exception (the normal distribution), all members of the family of symmetric stable Paretian distributions do not have finite variances or covariances. An interesting question is whether diversification has any meaning in a symmetric stable Paretian market. This problem was investigated by Fama (1965b, pp. 404-419).

Using the market model, and a theoretical scale parameter, Fama shows analytically that diversification is effective in reducing the dispersion or scale of a return distribution of a portfolio as long as the characteristic exponent, $c > 1$. Moreover, a given amount of diversification is more effective the higher the value of c. For $c = 1$, diversification is ineffective, and for $c < 1$, increasing diversification actually causes the dispersion of the portfolio return distribution to increase. Thus, for $1 < c \leq 2$, it was shown that diversification works and that beta remains a measure of systematic risk. Further, Fama's empirical work (Fama 1965a, pp. 34-105) indicates that for stocks of large American companies the value of c is most probably between 1.7 and 1.9. Fama concludes that diversification is an effective tool for reducing the dispersion of a portfolio return distribution.

Diversification: A Summary

The objective of this chapter was to define diversification formally. In connection with this objective, diversification within the context of the Markowitz model was examined. It was found that as portfolio size increased, the dispersion of the portfolio return distribution reduced to the average covariance among the securities in the portfolio. At this level, the variance of return of each security had no important effect on the variance of the portfolio return distribution. Thus, the risk of any security is measured by its marginal contribution to the average covariance of the portfolio's securities.

Efficient portfolios are those which satisfy the Markowitz rule. Portfolio analysis requires the determination of the efficient set of portfolios. This determination reduces to a quadratic programming problem requiring knowledge of E_p and V_p. The inputs required to solve the problem, making direct use of the Markowitz formulation, were $(N^2 + 3N)/2$. Because of the large number of inputs required by the Markowitz model, a simplified model developed by Sharpe, and requiring only $3N + 2$ inputs has gained prominence in portfolio analysis.

It was shown in this chapter that the simplified model (i.e., the market model) retains the essential features of the Markowitz portfolio theory. Using the market model, it was again shown that diversification resulted in reduction of the dispersion of the portfolio return distribution. Systematic risk, within the context of the market model, was defined to be beta, the slope of the linear relationship between R_{it} and R_{mt}. And, finally, using the market model, it was noted that diversification also works in a stable Paretian market if the characteristic exponent has a value greater than 1. Also, for this same range, beta remains well defined as the systematic risk.

CHAPTER 3

ANALYTICAL DISCUSSION

The objective of this chapter is to examine the prediction error problem analytically. First, the distributions that are considered in the analysis are identified, and reasons for their consideration are given. Then a formal definition of the prediction error problem is developed and the measures that are used in the analytical development are presented. Predictions imply the use of some prediction model. A particular prediction model is specified and reasons for the choice are given. Next, the cross-sectional independence assumption of the market model and its implications for the predicted betas are discussed. Finally, the effect diversification has on prediction error for each of the distributions identified earlier in the chapter is analyzed.

Return Distributions

Symmetric Stable Paretian

As noted in Chapter 1, only a certain class of return distributions satisfy the requirements of conventional capital market theory. This class of distributions has certain desired features. The distributions must be able to be completely specified by two parameters, namely, a measure of central location and a measure of dispersion. It is also necessary that the stability property holds. A particular set of distributions which has these desired features is the symmetric stable Paretian distributions.

It appears that actual return distributions may belong to the above class of distributions. There is considerable empirical evidence (Fama 1965a; Mandelbrot and Taylor 1967) that return distributions are symmetrical but have tails which are too "fat" for a normal distribution, i.e., extreme values occur more frequently than a normal distribution would predict. This fat tailed quality is displayed by the non-normal symmetric stable Paretian distributions. Because of this evidence, Fama (1965a) suggested that return distributions are members of the symmetric stable Paretian family. Moreover, as described in the last section of Chapter 2, the relevant distributions of this family, as far as portfolio theory is concerned, are those with values of the characteristic exponent in the interval $1 < c \leq 2$. Again, the empirical evidence (Fama 1965a) conforms with this requirement.

The statistical theory of symmetric stable distributions is discussed extensively in Gnedenko and Kolmogorov (1954, pp. 162-183). In this dissertation only the essentials necessary for the analytical development are discussed. Symmetric stable Paretian distributions are described by three parameters: a parameter of central location, m; a dispersion parameter, s; and a characteristic exponent, c. The characteristic exponent, c, determines total probability contained in the extreme tails of the distribution. It can assume any value in the interval $1 < c \leq 2$.

When $c = 2$, the relevant distribution is the normal distribution. When $c = 1$, the distribution is the Cauchy distribution. These are the only two members of the stable Paretian family which have known probability density functions. The general form of the logarithm of the characteristic function for the symmetric stable Paretian family of distributions is:

$$\log f(t) = \log E(e^{ixt})$$
$$= imt - s|t|^{c} \qquad (3.1)$$

where:

x = a random variable,

t = any real number,

$i = \sqrt{-1}$,

and m, s, and c are the three parameters previously defined. One other distributional possiblity, compatible with capital market theory, has recently been suggested in the literature. The distribution considerd is simply the normal distribution but with parameters which change over time.

The Contaminated Normal Distribution

Recent evidence (Boness, Chen and Jatsipitak 1974; Hsu, Miller and Wichern 1974) suggests that the fat tail phenomenon may result from a mixture of normal distributions (this mixture is labeled a contaminated normal). That is, the fat tails are due to a nonstationary Gaussian sequence of processes. Boness et al. (1974) view price relatives as following a random walk, with stable mean and variance. Some economic event occurs which causes changes in the parameters of the random walk, after which stability is again assumed. For example, adding or deleting a line of business may affect the parameters of the return distribution.

Boness et al. (1974) suggest that return distributions are normally distributed with discrete shifts in variance. Firms were studied before and after capital structure changes and the return distributions were found to have different variances in the two periods. Populations with different variances but the same mean would have thicker tails than a normal distribution.

Hsu et al. (1974) ran a chi-square goodness of fit test on the return distributions of several companies, hypothesizing symmetric stable Paretian distributions. They found that symmetric stable Paretian distributions were not consistent with the data. Further investigation indicated that the distributions were undergoing parameter shifts over time. In particular, the empirical data were not homogeneous with respect to variability. They concluded that the distribution of rates of return is nonstationary in the scale parameter over time, and that within subperiods of homogeneous behavior, normal distributions have adequate descriptive ability. Thus, current thought appears to be in favor of returns following a nonstationary Gaussian process. However, either of the two views is compatible with the distribution requirements of capital market theory (this statement is true because one is interested in predicting beta at a point in time). Thus, the effect of diversification on prediction error is examined within the context of each view of the return distributions. Before this analysis occurs, it is necessary to formally define the prediction error problem and specify the measures of prediction error that are used in the analysis.

Formal Treatment of Prediction Error

Prediction Error Defined

For purposes of defining prediction error, assume that β_{it+1} represents the true value of the systematic risk for the ith security for a future holding period ending at t + 1. Also assume investors use some prediction model that will produce an ex ante estimate of β_{it+1}. Let this estimate be $\hat{\beta}_{it}$. Prediction error, U_i, can then be defined as the difference between $\hat{\beta}_{it}$ and β_{it+1}, i.e., $U_i = \hat{\beta}_{it} - \beta_{it+1}$. In this context, note that prediction error arises because the true beta is an ex ante concept and must be estimated statistically.

Prediction error can be related to the portfolio level using the following notation:

$$\hat{\beta}_{1t} = \beta_{1t+1} + U_1$$
$$\hat{\beta}_{2t} = \beta_{2t+1} + U_2$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$\underline{\hat{\beta}_{Nt}} = \underline{\beta_{Nt+1}} + \underline{U_N}$$

$$\hat{\beta}_{pt} = \Sigma\hat{\beta}_{it}/N = \Sigma\beta_{it+1}/N + \Sigma U_i/N \tag{3.2}$$

where $\hat{\beta}_{pt}$ is the estimate of the risk of the portfolio. Prediction error at the portfolio level is defined as $\hat{\beta}_{pt} - \beta_{pt+1}$ or $\Sigma U_i/N$, where β_{pt+1} is the true portfolio beta at t + 1 and equals $\Sigma\beta_{i,t+1}/N$.

As can be seen from equation (3.2), the prediction of a portfolio beta involves an estimate of the beta of each security in the portfolio. The contention that prediction error can be diversified away is equivalent to stating that the last term on the right hand side of equation (3.2) approaches zero as N becomes arbitrarily large. That is, if this condition holds, then, for N large $\hat{\beta}_{pt}$ is very nearly equal to β_{pt+1} and prediction error has become inconsequential at the portfolio level via diversification. With the prediction error problem formally defined, the measures of prediction error that are used in the later analysis will now be presented and justified.

Analytical Measures of Prediction Error

Scale Parameter. Assume initially that $\hat{\beta}_{it}$ is a random variable and that $E(\hat{\beta}_{it}) = \beta_{it+1}$. Thus, $\hat{\beta}_{pt} = \Sigma\hat{\beta}_{it}/N$ is also a random variable and has a probability distribution centered on β_{pt+1}. The dispersion of this distribution results from $\Sigma U_i/N$. If diversification works then the dispersion will decrease as N increases. And, if the scale parameter approaches zero as N increases, then prediction error is eliminated at the portfolio level. Therefore, the scale parameter of the distribution of $\hat{\beta}_{pt}$ can be used to determine the impact of diversification on prediction error.

Expected Mean Square Error. If $E(\hat{\beta}_{it}) \neq \beta_{it+1}$ then a bias is introduced in the predictor, $\hat{\beta}_{it}$. For example, if the individual betas are nonstationary over time, bias in estimating each of these betas is likely to result. Nonetheless, it is still possible that the individual biases tend to cancel, and prediction error is inconsequential at the portfolio level. In the case of possible bias, the analytical measure that is used is the expected mean square error, which is defined as $E(\hat{\beta}_{pt} - \beta_{pt+1})^2$ or $E(U_p)^2$. And, if $\lim_{N \to \infty} U_p = 0$ then the $\lim_{N \to \infty} U_p^2 = \lim_{N \to \infty} U_p \lim_{N \to \infty} U_p = 0$. Thus, if the expected mean square error approaches zero as N increases, then one can conclude that prediction error has been eliminated by diversification. Prediction error, of course, implies the use of a prediction model. The prediction model used in this study will now be identified.

The Prediction Model

One important constraint of the prediction model is that it should not employ accounting data as independent variables since the issue being investigated is whether or not diversification obviates the need for such information. The prediction model which seems most appropriate in this context is a sum of linear terms fitted by OLS. However, in order to apply OLS, a model must exist to which it can be applied. It seems desirable that the model to which OLS is applied reflects the theoretical concepts inherent in the framework of capital market theory. The market model is a logical selection since it relates directly to concepts within capital market theory.

There are several reasons for the choice of OLS as the actual prediction mechanism. First, all of the assumptions of the market model, except for the cross-sectional independence assumption, are also the assumptions underlying OLS regression. Moreover, empirical work has shown that when OLS is applied to the market model, the assumptions of OLS are reasonably met (Blume 1968). Also, since beta is an ex ante concept and the future is unknown, beta must be measured from ex post return data; OLS is a computational procedure which allows the calculation of the market ex post beta (i.e., the beta as assessed from ex post market returns). And, finally OLS has been widely used in developing initial predictions of beta (Beaver et al. 1970; Rosenberg and McKibben 1973).

One other prediction model which receives limited application in the empirical section of this study is a Bayesian adjustment procedure suggested by Vasichek (1973). This procedure adjusts the observed betas toward the mean of the cross-sectional distribution of the sample of securities being studied. Bogue (1972) has shown this procedure to result in less prediction error than OLS, when prediction error is evaluated in terms of mean square error of predicting returns. It is possible that the Bayesian adjustment, when coupled with diversification could eliminate prediction error. The details of the procedure are discussed in Chapter 4. Having identified the prediction model, the independence assumption of the market model and its implications for the predicted betas are now examined.

The Issue of Independence

The market model assumes that the only source of covariance among securities is the market. If this assumption is true, then the cross-sectional independence assumption of the market model is met, i.e., $E(e_i e_j) = 0$, $i \neq j$. This result, in turn, would imply that β_i and β_j $(i \neq j)$ are independent. This can easily be shown. The OLS estimates of the parameters of the market model can be expressed in matrix form as:

$$\hat{B}_i = (X'X)^{-1}(X'Y_i) \qquad (3.3)$$

where

$$\hat{B}_i = \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix}, \text{ a 2x1 vector,}$$

X = an nx2 market return matrix,

Y = nx1 vector of returns of security i.

Taking the expected value of $\hat{B}_i$ for a given set of market returns:

$$E(\hat{B}_i) = (X'X)^{-1}X'E(Y_i) \ . \qquad (3.4)$$

For the covariance matrix of B_i and B_j, one has from equations (3.3) and (3.4):

$$\begin{aligned} E[\hat{B}_i-E(\hat{B}_i)][\hat{B}_j-E(\hat{B}_j)]' &= E\{[(X'X)^{-1}X'Y_i - (X'X)^{-1}X'E(Y_i)] \cdot \\ &\quad [(X'X)^{-1}X'Y_j - (X'X)^{-1}X'E(Y_j)]'\} \\ &= (X'X)^{-1}X'E\{[Y_i-E(Y_i)][Y_j-E(Y_j)]' \cdot \\ &\quad X(X'X)^{-1}\} \end{aligned} \qquad (3.5)$$

and, using equation (2.3) to substitute[1] for elements of Y_i, Y_j, $E(Y_i)$, and $E(Y_j)$, equation (3.5) simplifies to

$$E[\hat{B}_i - E(\hat{B}_i)][\hat{B}_j - E(\hat{B}_j)]' = (X'X)^{-1}X'E(u_iu_j')X(X'X)^{-1}$$

where u_i and u_j are vectors of residuals, each with n elements. But by the cross-sectional independence assumption of the market model, all elements of $E(u_iu_j')$ are zero. Whence,

$$E[\hat{B}_i - E(\hat{B}_i)][\hat{B}_j - E(\hat{B}_j)] = 0,$$

and, consequently,[2]

$$E[\hat{\beta}_i - E(\hat{\beta}_i)][\hat{\beta}_j - E(\hat{\beta}_j)] = 0 \ .$$

From the foregoing analysis, it is seen that if the cross-sectional independence assumption of the market model holds, then the estimated

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1. That is, each element of the vectors, Y_i and $E(Y_i)$, is replaced, respectively, by $\alpha_i + \beta_i E(R_{mt}) + e_{it}$ and $\alpha_i + \beta_i E(R_{mt})$, t = 1, . . . n. The same type of replacement also occurs for Y_j and $E(Y_j)$.

2. The weaker requirement that Covar $(\hat{\beta}_i, \hat{\beta}_j) = 0$ suffices for purposes of prediction error analysis. Of course, if normality is involved then Covar $(\tilde{\beta}_i, \hat{\beta}_j) = 0$ implies independence.

betas have no interrelationships. However, in reality, this assumption may not hold strictly. Thus, where possible, the effect that beta dependency has on prediction error will also be examined analytically. With this consideration, the background material for the analytical examination is complete. The prediction error analysis has two subdivisions -- one which examines the family of symmetric stable Paretian distributions and one which examines the nonstationary normal case. The symmetric stable is considered first.

Prediction Error Analysis

Stable Paretian Case

The question of interest is whether or not the effects of prediction error can be eliminated at the portfolio level. The answer to the question depends to some degree on the nature of the sampling distributions of the estimated betas. The first case considered is that of the symmetric stable Paretian distributions with stationary parameters. Estimating β_{jt+1} via OLS using a sample of past observations generates a sampling distribution for $\hat{\beta}_{jt}$. Assuming $f(R_{jt}|R_{mt})$ is symmetric stable Paretian, Wise(1966) demonstrates that the OLS estimators are distributed stable Paretian and are unbiased, but not efficient.[3] Since $\hat{\beta}_{jt}$ is an unbiased estimator, the scale parameter measures the prediction error present in the estimate, $\hat{\beta}_{jt}$. In other words, sampling variability is

3. Note that the analysis which follows only has two requirements. First, the predictor is assumed to be unbiased. Second, the predictor is assumed to be a random variable with a probability distribution which is symmetric stable Paretian. Any prediction model which produces predictors that satisfy these two requirements could be used.

the only source of prediction error. That is, $U_i = S_i$ and $U_p = S_i/N$, where S_i represents the sampling error in the estimate, $\hat{\beta}_{jt}$, and U_p is the aggregate effect of this error at the portfolio level.

The scale parameter of the sampling distribution of $\hat{\beta}_{pt}$ is used to assess the effect diversification has on prediction error at the portfolio level. This scale parameter is represented by s_p. If prediction error can be eliminated by diversification then s_p should approach zero as N gets large. The behavior of s_p as N increases can be determined by examining the log of the characteristic function of $\hat{\beta}_{pt}$. In order to specify this function, it is first necessary to determine the log of the characteristic function of the individual $\hat{\beta}_j$'s.[4] Since $\hat{\beta}_j$ is a random variable, the log of its characteristic function is:

$$\text{Log } f\hat{\beta}_j(t) = \text{Log } E(e^{i\hat{\beta}_j t}) = imt - s_j|t|^c$$

where m, s, and c are as defined in equation (3.1).

Of central interest is s_p, the measure of prediction error at the portfolio level. The estimated risk of the portfolio, $\hat{\beta}_p$, is equal to the weighted sum of the individual estimates, $\hat{\beta}_1/N + \hat{\beta}_2/N + \ldots$ $\hat{\beta}_N/N$. Assuming $\hat{\beta}_i$ and $\hat{\beta}_j$ $(i \neq j)$ are independent symmetric stable Paretian Random variables with the same characteristic exponent, then $\hat{\beta}_p$ will also be a symmetric stable Paretian by virtue of the stability property (i.e., a linear combination of symmetric stable Paretian random variables is also a symmetric stable Paretian random variable). Therefore, the log of the characteristic function of $\hat{\beta}_p$ is:

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4. To avoid confusion with the t in equation (3.1), the subscript t of $\hat{\beta}_{jt}$ is dropped.

$$\text{Log } f_{\hat{\beta}_p}(t) = \sum_{j=1}^{N} \text{Log } f_{\hat{\beta}_j}(t/N)$$

$$= \sum_{1}^{N} i(m/N)t - \sum_{1}^{N} s_j (1/N)^c |t|^c$$

The location and scale parameters of the sampling distribution of $\hat{\beta}_p$ are:

$$m_p = \Sigma\, m_j/N \qquad (3.5)$$

$$s_p = \Sigma\, (1/N)^c s_j \qquad (3.6)$$

The behavior of s_p in equation (3.6) as N increases depends on the value of c, the characteristic exponent. Specifically, when $o < c \leq 1$, s_p does not decrease, and, in fact, approaches either the average value of the individual s_j's or increases without bound. In explicit terms, for c = 1, as N increases

$$\lim_{N \to \infty} s_p = \lim_{N \to \infty} (1/N)\, \Sigma s_j$$

$$= \lim_{N \to \infty} \bar{s}$$

$$= \bar{s}$$

and, for $0 < c < 1$, as N increases,

$$\lim_{N \to \infty} s_p = \lim_{N \to \infty} (1/N)^c\, \Sigma s_j$$

$$= \lim_{N \to \infty} (1/N)^{c-1}\, \bar{s}$$

$$= \infty\ , \text{ as } c - 1 < 0\ .$$

However, for $1 < c \leq 2$, s_p decreases with an increasing N, and, approaches zero. That is,

$$\lim_{N \to \infty} s_p = \lim_{N \to \infty} (1/N)^c \Sigma s_j$$

$$= \lim_{N \to \infty} (1/N)^{c-1} \bar{s}$$

$$= 0 \quad .$$

Thus, diversification is effective in eliminating prediction error only in the case where $1 < c \leq 2$.[5] As previously mentioned, Fama (1965a) has provided empirical evidence that c for return distributions is most probably between 1.7 and 1.9. Acceptance of this evidence results in the conclusion that diversification can eliminate prediction error at the portfolio level.

The above development assumes $\hat{\beta}_i$ and $\hat{\beta}_j$ are independent. If independence does not exist, then in addition to the dispersion component on the right side of equation (3.6) there would be a codispersion component. In other words, the sampling distribution of $\hat{\beta}_p$ is influenced by the dispersion of the sampling distributions of each $\hat{\beta}_i$ and by the dispersion caused by the interrelationships of the estimated betas. Unfortunately, a precise mathematical statement of the above intuitive notion is available for only one member of the symmetric stable Paretian family. Little is available in the literature concerning the characteristics and behavior of dependent stable Paretian random variables.

However, for the one member mentioned above, the analysis can be done. This member is the normal distribution. For c= 2, normality

5. The proof presented above parallels the one developed by Fama (1965b) concerning diversification and individualistic risk in a stable Paretian market.

holds and it is possible to talk about covariance, a well defined measure of codispersion. For this case,

$$\mathrm{Var}(\hat{\beta}_p) = (1/N^2)\ \Sigma\ \mathrm{Var}(\hat{\beta}_i) + (1/N^2) \sum_{\substack{i\\ i\neq j}} \sum_{j} \mathrm{Covar}\ (\hat{\beta}_i, \hat{\beta}_j) \tag{3.7}$$

and

$$\lim_{N \to \infty} \mathrm{Var}(\hat{\beta}_p) = \lim_{N \to \infty} \overline{\mathrm{Var}\ (\hat{\beta})}/N + \lim_{N \to \infty} [N(N-1)/N^2]\ \overline{\mathrm{Covar}}$$

$$= \overline{\mathrm{Covar}} \ .$$

For this member of the stable family, prediction error is not eliminated but approaches the average covariance among the estimated betas. If the dependency is mild, it may be the above result has no practical importance. However, this possibility would have to be investigated empirically.

An interesting speculative result exists relative to the above analysis. It may be that the structure of equation (3.7) holds for the other members of the stable family. That is, replacing the variance, covariance, and the number 2 with s_i, s_{ij}, and c, respectively, the result is:

$$s_p = (1/N)^c\ \Sigma\ s_i + (1/N)^c \sum_{\substack{i\\ i\neq j}} \sum_{j} s_{ij} \tag{3.8}$$

where:

s_{ij} = a theoretical measure of codispersion,

and, as N increases,

$$\lim_{N \to \infty} s_p = \lim_{N \to \infty} (1/N)^c \, \Sigma \, s_i + \lim_{N \to \infty} (1/N)^c \sum_{i} \sum_{\substack{j \\ i \neq j}} s_{ij}$$

$$= \lim_{N \to \infty} [N(N-1)/N^c] \, \bar{s}_{ij}$$

$$= \lim_{N \to \infty} \frac{1}{(c-1)N^{(c-2)}} \, \bar{s}_{ij}$$

which, for $c < 2$,

$$= \infty$$

If the structure of (3.7) were to hold, then prediction error would increase without bound.

Contaminated Normal -- Nonstationary Beta Case

If return distributions are normal with nonstationary parameters, then it is likely that the betas of securities also are nonstationary. In fact, as Downes and Dyckman (1973, p. 311) note:

> Logic, as well as the empirical results, tells us to expect changes in these coefficients because of such events as changes in tastes, mergers, the effect of changing economic conditions on growth firms, the effect of changes in the debt-equity ratio caused by new issues, debt retirements, or shifts in the level of a stock's price, and to governmental policies in the areas of trade, taxes, and so on.

As implied by the above quote, empirical evidence exists indicating that betas are nonstationary over time.

Boness et al. (1974) studied 33 firms before and after capital structure changes. Using the Chow test, they found that the betas of the majority of the firms were significantly different across the two subperiods. Abner (1972) also examines stability of betas. Stability is inferred by the ability of beta values derived from different time periods, using regression techniques, to produce similar distributions of returns when applied to identical market index values. He concluded from his tests that there exists a low degree of parameter stability. Other studies resulting in the same kind of conclusions are Blume (1971), Meyers (1973), and Hinich and Roll (1975).

With beta undergoing structural changes over time, an additional type of prediction error on β_{it+1} is introduced when historical data are used in the prediction model. It is possible, when estimating beta for a future holding period, that beta has changed from what it was in the past or that past data are already composed, as is likely, of different relationships. In either case, a bias due to nonstationarity results. Under the assumption of stationarity $E(\hat{\beta}_{it}) = \beta_{it+1}$; however, if nonstationarity holds then $E(\hat{\beta}_{it}) = \beta_{it+1} + V_i$, where V_i is the nonstationary bias. Thus, under nonstationarity, prediction error on an individual beta is equal to the sum of the sampling error and the nonstationary bias (i.e., $U_i = S_i + V_i$). And, at the portfolio level, prediction error is

$$U_p = \Sigma S_i/N + \Sigma V_i/N \tag{3.9}$$

The results of this kind of prediction error can now be analyzed. Hinich and Roll (1975) suggest that a possible advantage of portfolio formation is to smooth out changes in the model's parameters which are unique to individual firms. Studies by Blume (1971) and Levy (1971) are often quoted as evidence that portfolio betas are relatively stable and, consequently, as evidence that the nonstationary effects of individual securities vanish at the portfolio level. The evidence is that the betas of several portfolios in one time period are highly correlated with the betas of the same portfolios in a subsequent time period. Downes and Dyckman (1973, p. 311) make the following observation:

> The importance of the stability of the individual betas declines . . . since their relevance to portfolio risk is their contribution to the average squared portfolio beta. It may be possible even in high-(low) risk strategy portfolios to diversify out of some (much?) of the effect of shifting beta values for individual securities held. Also for portfolios of relatively small size, and involving a moderate risk strategy, movements of individual betas will tend to offset one another.

And, in a similar vein, Lorie and Hamilton (1973, p. 224), in speaking of the problem of nonstationarity of individual betas, note that:

> The seriousness of this fact is not great when one realizes that one is interested in betas for portfolios rather than for their component assets. The law of large numbers helps somewhat. Estimates of beta are sometimes too high and sometimes too low. These discrepancies are partially offsetting with the result that estimates for portfolios are often quite good predictors of future betas for portfolios.

The thrust of all of the above is the belief that the effects of nonstationary bias are not felt at the portfolio level. More formally, the belief is equivalent to stating that $\Sigma V_i/N \rightarrow 0$ as N becomes large. That is, diversification eliminates any prediction error that could be caused by shifting beta values. Thus, the view is that the nonstationary effects cancel out by diversification, leaving a portfolio beta which remains stable over time. With this understanding, the effects of individual beta nonstationarity at the portfolio level will now be examined.

Nonstationary Effects

To investigate the impact of nonstationary effects on prediction error, assume initially, that $\hat{\beta}_{it}$ and $\hat{\beta}_{jt}$ ($i \neq j$) are independent. The measure of prediction error used in the analysis is the expected mean square error, which is designated P_e. The analysis which follows shows that P_e does not vanish as N increases, but rather approaches the average value of the cross product of the nonstationary biases. First, consider P_e for an individual security:

$$P_e = E(\hat{\beta}_{it} - \beta_{it+1})^2 = E[\hat{\beta}_{it} - E(\hat{\beta}_{it}) + E(\hat{\beta}_{it}) - \beta_{it+1}]^2$$

$$= E[\hat{\beta}_{it} - E(\hat{\beta}_{it})]^2 + E[E(\hat{\beta}_{it}) - \beta_{it+1}]^2$$

$$= \mathrm{Var}(\hat{\beta}_{it}) + V_i^2 \; . \tag{3.10}$$

And, at the portfolio level:

$$P_e = E(\hat{\beta}_{pt} - \beta_{pt+1})^2 = E[1/N(\hat{\beta}_{1t} - \beta_{1t+1}) + 1/N(\hat{\beta}_{2t} - \beta_{2t+1}) + \ldots (1/N)(\hat{\beta}_{Nt} - \beta_{Nt+1})]^2$$

$$= (1/N^2) \sum_{1}^{N} E(\hat{\beta}_{it} - \beta_{it+1})^2 + (1/N^2) \sum_{i} \sum_{\substack{j \\ i \neq j}} E(\hat{\beta}_{it} - \beta_{it+1}) \cdot (\hat{\beta}_{jt} - \beta_{jt+1}) \tag{3.11}$$

Substituting (3.10) into the first term on the right hand side of (3.11), and noting that $E(\hat{\beta}_{it} - \beta_{it+1})(\hat{\beta}_{jt} - \beta_{jt+1}) = V_i V_j$, the result is:

$$P_e = (1/N^2)\ \Sigma\ \mathrm{Var}(\hat{\beta}_{it}) + (1/N^2)\ \Sigma\ V_i^2 + (1/N^2) \sum_{\substack{i\ j \\ i \neq j}} \Sigma\ V_i V_j\ . \quad (3.12)$$

And, as the portfolio size increases,

$$\lim_{N \to \infty} P_e = \lim_{N \to \infty} [\overline{\mathrm{Var}}(\hat{\beta})]/N + \lim_{N \to \infty} \frac{\bar{\bar{V}}^2}{N} + \lim_{N \to \infty} [(N-1)/N]\ \overline{V_i V_j}$$

$$= \overline{V_i V_j}\ .$$

Thus, even when independence is assumed, some nonstationary effects remain at the portfolio level. Of course, if the magnitude and direction of V_i and V_j are random, it is conceivable that $\overline{V_i V_j}$ will not be of consequence because of cancelling effects. However, such possibilities must be dealt with empirically.

If the assumption of independence is not met, then the following analysis demonstrates that prediction error at the portfolio level has an additional component. In equation (3.11) $E(\hat{\beta}_{it} - \beta_{it+1})(\hat{\beta}_{jt} - \beta_{jt+1})$ is no longer equal to $V_i V_j$ but is equal to $V_i V_j$ + Covar $(\hat{\beta}_{it}, \hat{\beta}_{jt})$ since:

$$E(\hat{\beta}_{it} - \beta_{it+1})(\hat{\beta}_{jt} - \beta_{jt+1}) = E\{[\hat{\beta}_{it} - E(\hat{\beta}_{it}) + E(\hat{\beta}_{it}) - \beta_{it+1}]$$
$$\cdot\ [\hat{\beta}_{jt} - E(\hat{\beta}_{jt}) + E(\hat{\beta}_{jt}) - \beta_{jt+1}]\}$$

which equals,

$$E\{[\hat{\beta}_{it} - E(\hat{\beta}_{it})][\hat{\beta}_{jt} - E(\hat{\beta}_{jt})] + V_i[\hat{\beta}_{jt} - E(\hat{\beta}_{jt})] + V_j[\hat{\beta}_{it} - E(\hat{\beta}_{it})] + V_i V_j\}$$

and, finally, running the expected value operator through this last expression yields $V_i V_j$ + Covar $(\hat{\beta}_{it}, \hat{\beta}_{jt})$. Thus, equation (3.12) takes the modified form:

$$P_e = \overline{Var}(\hat{\beta})/N + \bar{V}_i^2/N + [(N-1)/N][\overline{Covar} + \overline{V_i V_j}] ,$$

and, as $N \to \infty$, $P_e \to \overline{Covar} + \overline{V_i V_j}$.

The additional component is the average covariance among the betas. It does not appear that diversification is effective in the dependency case. However, it may be that the dependency is mild and the additional component (i.e., $\overline{Covar}$) may not be of practical importance. Again, empirical investigation is required to assess the impact of the violation of the independence assumption.

Summary

The objective of this chapter was to examine the prediction error problem analytically. First, the necessary background material for the analysis was developed. The first topic of this development was a discussion of the probability distributions considered in the analysis. The family of symmetric stable Paretian distributions and the contaminated normal were discussed. These distributions were chosen because they were consistent with both the requirements of capital market theory and available empirical evidence.

Next, a formal definition of the prediction error problem was presented. Basically, mathematical structure was given to the claim that diversification eliminates prediction error. Also, within this section of the chapter, the prediction error measures used in the analytical development were defined. These measures were the scale parameter of the sampling distribution of the beta predictors and the expected mean square error. It was shown that these measures properly indicate the elimination of prediction error if diversification works.

The next step in the development of the background material was the specification of the prediction model. The actual prediction mechanism was identified as OLS. The linear model to which OLS is applied was identified as the market model. Reasons for the above choices were given and were related to the requirements of capital market theory as well as practical considerations.

The issue of independence was discussed because of its potential impact on prediction error analysis. It was shown that the beta predictors are not related if the cross-sectional independence assumption of the market model is valid. However, because of the possibility that this assumption is not strictly true, it was decided to also analytically examine prediction error effects under dependency.

The final section of the chapter consisted of the actual analytical development of the prediction error problem. This section had two major areas of consideration. First, prediction error was examined assuming that the sampling distributions of the beta predictors were symmetric stable Paretian. Assuming independence, it was shown that prediction error could be eliminated by diversification. For the case of dependency, because little information exists concerning dependent stable Paretian random variables, only one member of the stable family could be analyzed. However, for the one member (i.e., the normal distribution) it was found that diversification did not eliminate prediction error. The residual prediction error at the portfolio level was equal to the average covariance among the estimated betas.

Finally, it was assumed that the sampling distributions of the beta predictors were normal with nonstationary parameters. Specifically, it was assumed that beta changes over time. Under independence, it was demonstrated that prediction error approached the average cross product of the nonstationary biases. Dropping the assumption of independence resulted in an additional error component remaining at the portfolio level, namely, the average covariance among the predicted betas.

The analysis of this chapter raises some serious doubts as to whether, in reality, diversification can eliminate prediction error. Specifically, this uncertainty is evident for the contaminated normal-nonstationary beta case, which recent empirical evidence indicates is the most likely of the possible cases. Because of this uncertainty, it was decided to examine the effects of prediction error at the portfolio level empirically under the assumption that the contaminated normal-nonstationary beta case holds. Chapters 4, 5, and 6 of this study are concerned with this empirical investigation.

CHAPTER 4

EMPIRICAL DESIGN AND METHODOLOGY

The purpose of this chapter is to describe the design of the empirical study and the means by which this design was accomplished. The first several sections of this chapter deal with topics that are necessarily preliminary to the actual formation of the portfolios used in the empirical study. The first section describes the selection of securities which are possible candidates for membership in the various portfolios. Next, the criteria for membership in these portfolios are outlined. The following section then gives a brief description of a number of statistical tests which are used to identify certain characteristics of a security and thus determine if the selection criteria are satisfied. The application process of these statistical tests is described and the results of the application are given. The final section of this chapter is concerned with the actual formation of the portfolios that were studied empirically.

Sample Selection

The sample of this study consisted of 197 New York Stock Exchange (NYSE) securities. The January 10, 1975 Compustat manual served as the population source. The sample was taken subject to the restriction that each firm had continuous daily returns on the Center for Research on Stock Prices (CRSP) tape from November 10, 1967 to July 11, 1975.[1] In addition to the above restriction, to insure that firms from a wide variety of industries were represented, the population was stratified on the basis of the first digit of the Compustat industrial code, and securities were randomly selected from each stratum.[2] A list of the firms selected and the industries they represent is given in Appendix A.

The daily returns were converted to monthly returns (i.e., a month equals 20 trading days). The interval, November 10, 1967 to July 11, 1975 contained 100 monthly returns. Return on a security is defined as:

$$\ln\,[(P_{it+1} + D_{it+1})/P_{it}]$$

where

P_{it} = price of the stock in period t,

P_{it+1} = price of the stock in period t+1,

D_{it+1} = cash dividends paid in period t+1.

Given the sample of securities, a decision must be made on how to place the securities in the different portfolios.

Selection Criteria for Membership in the Portfolio Sets

The criteria discussed in this section determine whether a security qualifies for membership in either or both of two sets of

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1. The selection of this time period was arbitrary except for two considerations. First, the length of the period allows the computation of 100 monthly observations. The computer program containing the statistical tests (purpose described later) restricts the input to a maximum of 100 observations. The second consideration is the belief that a length of 100 observations is sufficient to allow for structural changes in the betas of securities. The basis of this belief relates to the results of a pilot sample of three securities. These three securities were studied for an interval of 100 observations and two of the securities gave evidence of structural changes in their betas.

2. Diversified portfolios contain securities from a variety of industries (Lorie and Hamilton 1973, p. 270).

portfolios. Discussion of criteria for membership in specific portfolios of the two sets is deferred to the portfolio formation section of this chapter. The selection criteria follow essentially the same pattern established in the analytical discussion of prediction error (assuming the contaminated normal-nonstationary beta case).

The analytical discussion assumed that the beta of security i was stationary in period t + 1. For the contaminated normal case, it was assumed that the beta for security i in period t was structurally different from the beta of period t + 1. The securities of one of the two sets of portfolios follow this pattern with one modification. In reality, it is likely that a given portfolio has securities of two types. The first type are those securities with betas that are structurally different across the two time periods, t and t + 1. The second type are those securities with betas that remain the same across the two time periods. To allow for this possibility, both types of securities are permitted membership in this first set. The criteria for the first set of portfolios can now be specifically delineated.

For the first set of portfolios, securities are identified which have betas that are stationary in period t + 1 and which have betas in period t that are either structurally different or the same as the betas of period t + 1. Obviously, in order to evaluate the effect of the nonstationary bias component of prediction error, a substantial proportion of the securities in the portfolios of this set must have structurally different betas across the two periods. This requirement can be met by selecting the time interval so that it is sufficiently long to allow for structural changes in the betas of the securities.

Criteria for membership in the second set of portfolios differ in only one respect from the criteria of the first set. For the second set of portfolios, securities are identified which have betas that are stationary in period t + 1 and which have betas in period t that are the same as the betas of period t + 1. Thus, the only difference between the two sets is the presence of the nonstationary element in the first set (i.e., securities are allowed membership in the first set that have structurally different betas across the two time periods). Since the time periods can be different for the two sets, it is possible that a given security may belong to both sets. With the criteria identified, the discussion can now center on the process whereby securities are identified that satisfy the selection criteria.

Statistical Tests for Detecting Nonstationarity

Placing securities in each of the two sets requires the identification of the occurrence of nonstationarities. A computer program, called TIMVAR, and the Chow test (Chow 1960) were used to accomplish this objective. TIMVAR embodies a set of techniques for detecting departures from constancy of regression relationships over time when regression analysis is applied to time series data (Brown, Durbin and Evans 1973). More specifically, TIMVAR is used to test the hypothesis of constant regression coefficients over time,

$$H_o: \beta_1 = \beta_2 = . . . \beta_T = \beta$$

(β is used here to represent a vector of regression coefficients) where the subscript t = 1, . . ., T is used to indicate that β_i may vary with time.

There are four techniques used in the program. First, the above hypothesis can be investigated by constructing forward and backward (represented in later tables by FW and BW) plots of cumulative sums and sums of squares of recursive residuals. These two tests, cusum and cusum of squares, involve a pair of significance lines, which if crossed by the sample path, results in the rejection of the null hypothesis of constant regression coefficients.

The third technique is based on plotting the coefficients obtained by fitting the model to a segment of n successive observations and moving this segment along the time series. The plots are supplemented by a homogeneity test based on the analysis of variance. The final TIMVAR technique is the plotting of Quandt's log-likelihood ratio (Quandt 1958) which detects the single time point, if any, at which there is an abrupt change from one constant set of parameters to another.

The Chow test is used to determine if the set of coefficients of two separate regression lines is significantly different. Thus, this test was used to determine if the betas of the two periods were structurally different. A more detailed description of the Timvar and Chow techniques is given in Appendix B.

The TIMVAR program provides the results of the above tests, performs standard regression, and also provides a considerable amount of graphical output. The examination of the data from these several viewpoints allowed the selection of the securities in accordance with the criteria discussed in the previous section. This selection process is described in the next section.

Selection Process for the Portfolio Sets

Essentially, three computer runs on 197 securities were needed to identify time intervals where the betas satisfied the criteria previously outlined. On the first run, TIMVAR received an input of 100 monthly returns (hereafter referred to as observations) for each security. Two moving regression lengths of ten observations (designated as MR1) and fifteen observations (designated MR2) were used. The results of the TIMVAR tests for all three computer runs are summarized in Table 4.1.

For the first run, 172 securities displayed significance at the .05 level for at least one of the TIMVAR tests. The number of securities that were significant at a level of .05 for the cusum test (either forward or backward), cusum of squares test (either forward or backward), and the homogeneity tests were, respectively, 20, 144, 23, and 22. The results do not sum to 172 because some securities are significant for only one test, some for only two tests, etc.; in other words, there is overlap.

Only 25 of the 197 securities failed to show significance for at least one of the tests. The betas of these securities were classified as stable, since the results of the statistical tests were consistent with stationarity. Since these 25 securities qualified for both sets of portfolios, regardless of the eventual time period specifications, they were exempted from further consideration. The remaining 172 securities qualified for a second run, since identification of

Table 4.1. TIMVAR Results for Each Computer Run.

Run	Cusum FW or BW	Cusum Sq FW or BW	MR1	MR2	Number Intervals Significant
First	20	141	23	22	172[a]
Second	14	69	25	17	101[a]
Third	13	45	13	16	61[a]

a. This number represents the number of intervals that were significant for at least one of the TIMVAR tests. It does not represent the sum of the number of tests that were significant since some intervals were significant for only one test, some only for two tests, etc.

appropriate time intervals for these securities still had to be accomplished. Step one of Figure 4.1 illustrates, diagramatically, the selection procedures for this first run. The remaining two steps of Figure 4.1 help clarify the description of the selection procedures for the second and third runs.

As mentioned above, and as illustrated by Figure 4.1, 172 securities entered the second run. Since all of these 172 securities showed statistical significance for at least one of the TIMVAR tests, the point where Quandt's log-likelihood ratio achieved its minimum was used as the initial estimate of the point of disturbance. In most cases (128 out of 172 securities) this point agreed with the maximum deviation point of the cusum of squares sample path. Using the disturbance point as the point of division, the 100 observation interval was divided into two subintervals and a second run of TIMVAR was made on each of the subintervals.

If the subintervals displayed no evidence of nonstationarity then the location of a change in the security's beta was identified as the initial estimate (i.e., the division point). Eighty-seven of the 172 securities displayed this type of behavior (see step 2 of Figure 4.1). However, since the cusum of squares test[3] is sensitive to changes in the residual variance as well as to changes in the regression coefficients (Brown et al. 1973, p. 153) an F test on the equality of variances was executed on the subintervals of these 87 securities. The hypothesis tested was:

3. The TIMVAR test most frequently significant in all three runs was the cusum of squares test. See Table 4.1.

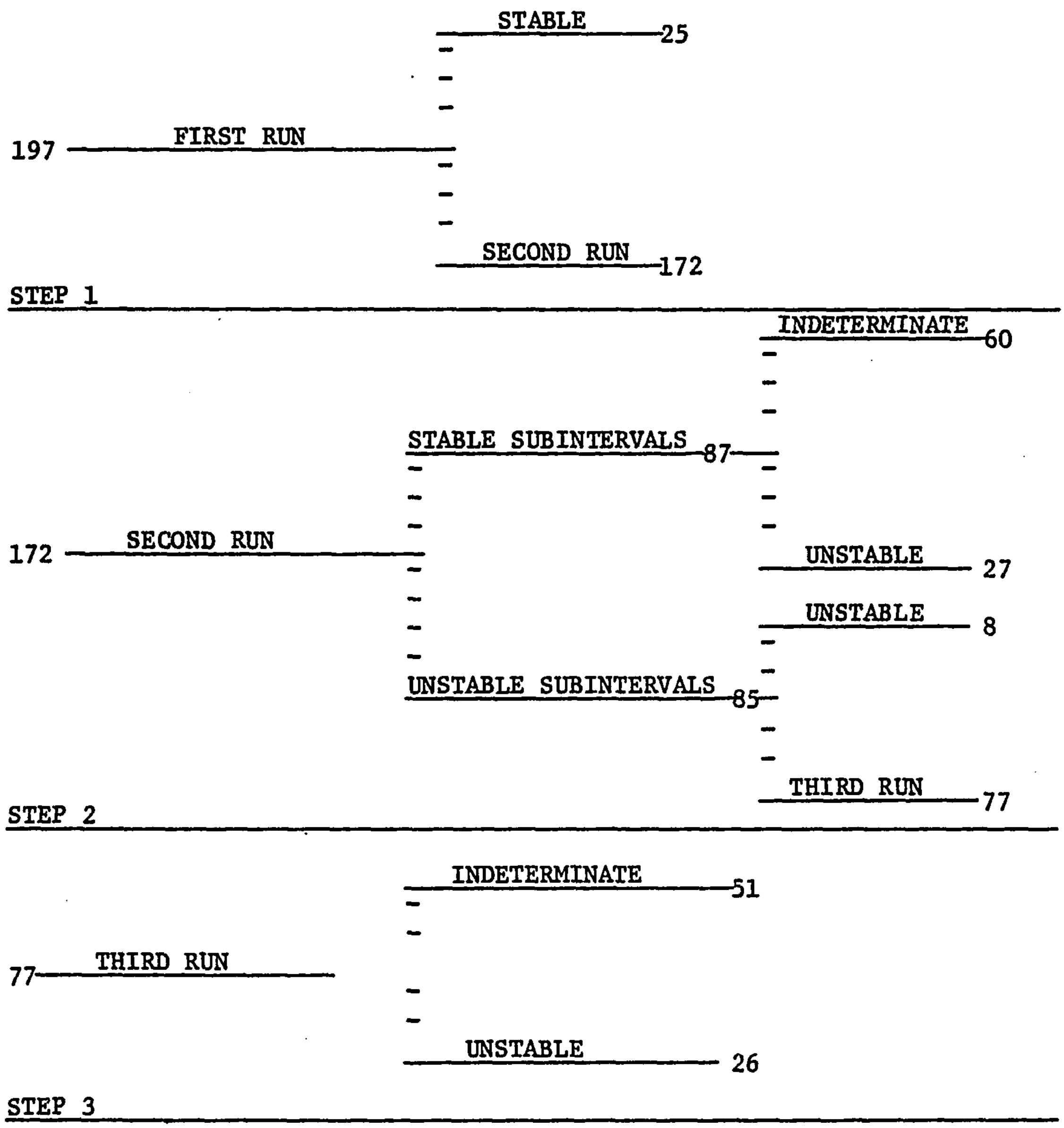

Figure 4.1. Diagram of the Selection Process.

$$H_o: V_{i1} = V_{i2}, i = 1, . . ., 87$$

where

V_{i1} = residual variance of security i in subperiod one,

and

V_{i2} = residual variance of security i in subperiod two.

The F test showed significance for 71 of the 87 securities at the .05 level. This left 16 securities as having test results consistent with constant variances.

Next, the Chow test was run for all 87 securities. The hypothesis examined by the Chow test is:

$$H_o: \beta_{i1} = \beta_{i2} ,$$

where

β_{i1} = the true beta of security i in subperiod one,

β_{i2} = the true beta of security i in subperiod two.

The Chow test was significant at the .05 level for 15 of the 16 securities with constant variance and with 11 of the 71 securities with non-constant variance. On the basis of the above results, it was decided to accept 27 securities (the 16 with constant variance plus the 11 with nonconstant variance referred to in the results of the Chow test) as having unstable betas and to view the remaining 60 as indeterminate since the TIMVAR results could be due primarily to a shift in residual variance and not to a change in beta (see the upper branch of step two in Figure 4.1).

There were 85 securities of the 172 which had one or more sub-intervals displaying nonstationary behavior (see Table 4.1 for the total number of intervals with significant TIMVAR results). Of these 85 securities, 28 had the second subinterval displaying evidence consistent with stable beta behavior. Upon running the Chow test for these 28 securities, the hypothesis that the betas were the same across the two subintervals was rejected in eight cases at a level of .05 significance. Thus, having identified an interval in which beta was nonstationary and a subsequent interval in which the evidence was consistent with stationarity, these eight securities were added to the previous 27 as having unstable betas. As the lower branch of step two of Figure 4.1 illustrates, subtracting the eight securities from the 85 securities with unstable subintervals leaves 77 securities which qualified for the third computer run.

The remaining 77 securities, the subintervals of which displayed significance for at least one of the four TIMVAR tests, were again subdivided using the same criterion for the division point as was used for the second run. A third run was then executed on these subintervals. Using the same procedures described for the previous two runs, another 26 securities were identified as having unstable betas within the 100 month interval. The remaining 51 securities were classified as indeterminate (see step three of Figure 4.1).

Table 4.2 summarizes the essential details of the three computer runs. This table presents the number of securities involved in each run, the number of time intervals to which TIMVAR was applied, the number of intervals which had at least one significant test, and the categorization of the betas of the securities as either stable, unstable, or indeterminate for the 100 month interval. From Table 4.2, it can be

Table 4.2. Analysis of Classification Process by Computer Run.

Run	No. Securities	No. Intervals	No. Intervals Significant	No. Unstable	No. Indet.	No. Stable
First	197	197	172[a]	--	--	25
Second	172	344	101[a]	35	60	--
Third	77	186	61[a]	26	51	--
TOTAL	--	--	--	61	111	25

a. Significant at a level of .05.

seen that within the 100 month period, 61 securities were identified as having unstable betas, 25 securities displayed evidence consistent with stable betas, and the betas of the remaining 111 securities were classified as indeterminate. An illustrative example of the selection process is given in Appendix C. With the above analysis complete, a search was made for time intervals that would satisfy the structure of the two sets of portfolios described in an earlier section of this chapter. The results of this search are reported in the next section.

Portfolio Formation

For the first set of portfolios, it was necessary to identify securities which had either stable or unstable betas for a time period t and stable betas for a subsequent time period, t + 1.[4] Of the 197 securities, 153 were identified as having behavior consistent with stable betas for a holding period of 14 consecutive months, June 14, 1974 to July 11, 1975. And, a minimum of 49 of these 153 securities were identified as having nonstationary betas within a prediction period of 86 months, November 10, 1967 to June 14, 1975. The word, "minimum," is used since 79 of the 153 securities in the prediction period belonged to the indeterminate category. Since these 153 securities satisfied the criteria for membership in the first set, they were used to form the first set, with 86 observations in the prediction period and 14 observations in the holding period. Because of the large

4. Throughout the remainder of this study, period t is referred to as the prediction period and period t + 1 as the holding period. Observations from period t are used to predict the beta of period t + 1, hence, prediction period. Period t + 1 is viewed as the future period of time the portfolio is held by an investor -- hence, holding period.

number of observations available in the prediction period to estimate each beta, there will be only a small amount of sampling error on each beta. Thus, the main potential source of prediction error for this set is nonstationary bias; therefore, this set can be used to evaluate the ability of diversification to eliminate the nonstationary component of prediction error. The first set, from hereon, is identified by the phrase, "nonstationary set."

For the second set of portfolios, it was necessary to identify securities which had stable betas for the prediction and holding periods. There were 90 securities that were identified as having betas consistent with stability for the first 30 months (of the 100 month interval), November 10, 1967 to February 27, 1970. These securities were then used to construct the second set of portfolios, with 15 observations in the prediction period and 15 observations in the holding period. Note that the minimum number of securities that have nonstationary betas in this set of portfolios is zero. The word, "minimum," allows for the possibility that some securities with nonstationary betas, may, in fact, enter the portfolios of the second set. This possibility exists because of the presence of the unknown type two error in hypothesis testing. Nonetheless, the nonstationary effects will be minimized at the portfolio level in this second set. Since the main source of prediction error for this set is sampling error, it can be used to evaluate the ability of diversification to eliminate this type of prediction error. The second set is labeled, "stationary approximation."

Only these two sets of portfolios are studied empirically. Two other configurations are possible. Portfolios could have been created that had a mixture of stable and unstable betas in both the prediction and holding periods. Also, portfolios could have been created that had stable betas in the prediction period and a mixture of stable and unstable betas in the holding period. However, it is felt that these two configurations would add little to the analysis. If it can be shown, using the nonstationary set, that nonstationary effects are not felt at the portfolio level, then the portfolio beta is stable over time. This result would reduce the remaining three configurations to a stable portfolio beta-stable portfolio beta configuration, in which case the only source of any remaining prediction error at the portfolio level is sampling error. But the stationary approximation set examines the ability of diversification to eliminate sampling error. Thus, if prediction error is eliminated by diversification for the nonstationary and stationary approximation sets, it is also eliminated for the other two possible configurations. With this consideration, the discussion now turns to the specification of the types of portfolios considered in the two sets of interest.

Types of Portfolios within the Two Sets

For both sets of portfolios, two separate estimates of the portfolio beta were obtained. One estimate was calculated using data from the prediction period, and the second was calculated using data from the holding period. The end point of the prediction period is viewed as the decision point. That is, this point is where an assumed investor determines the makeup of his portfolio for the coming holding period.

At this point, the investor needs to assess the risk of the portfolio, and, consequently, the risk of each security.

Several types of portfolios were formed based upon the prediction period betas. The securities in each of the two sets were arranged in descending order with respect to the magnitude of their betas. The top, middle, and lower thirds of the securities were then used to form three portfolios of differing risk. These three portfolios are named, respectively, high risk, medium risk, and low risk. In tables appearing in this study, these portfolios are identified by the symbols H, M, and L. Also, for each set, three portfolios were created by random assignment. These portfolios are named R1, R2, and R3. For the nonstationary set there are 51 securities in each of these six portfolios; the stationary approximation set has 30 securities in each of the six portfolios.

Of course, the portfolios are formed in order to investigate prediction error at the portfolio level. Supposedly, as the number of securities in a portfolio increase, prediction error decreases. To determine if this type of behavior occurs, the high risk portfolio and one of the randomly created portfolios (R1) each served as bases to which were added increments of 15 securities and 17 securities for the stationary approximation and nonstationary sets, respectively.

In the case of the high risk base, the increments first come from the medium portfolio, in order of descending magnitude of betas, and, finally, from the low risk portfolio in the same manner. The increments to the random based portfolio were randomly selected from the other two randomly created portfolios. The incremental portfolios are identified by Hi-i and R1-1 where i means the ith increment has been added. This procedure resulted in adding 11 additional portfolios to the nonstationary set and seven additional portfolios to the stationary approximation set (for the stationary approximation set R1-4 and Hi-4 are the same portfolio; R1-6 and Hi-6 are the same for the nonstationary set). Thus, the total number of portfolios considered for the stationary approximation set is 13 and for the nonstationary set is 17. Table 4.3 gives a summary of the types of portfolios and their sizes for each set of portfolios. A list of securities by firm and industry for each portfolio of the two sets is given in Appendix D. Also, for the nonstationary set the securities with unstable betas are indicated.

Synopsis of the Chapter

The purpose of this chapter was to describe the methodology and design of the empirical study of prediction error. First, it was noted that a stratified sample of 197 NYSE securities was taken. The sample was stratified on industry and subjected to the constraint that each security had continuous observations for a specified time period.

Next, selection criteria for membership in the two sets of portfolios were specified. One set required securities with stable betas for two consecutive time periods. The second set of portfolios required securities to have stable betas in the second time period, and allowed securities with either stable or unstable betas for the first of the two consecutive time periods.

Several statistical tests, capable of identifying nonstationarities of regression coefficients were described. There were essentially five procedures, namely, the cusum test, cusum of squares test,

Table 4.3. Types of Portfolios and Their Size for the Stationary Approximation and Nonstationary Sets.

	Stationary Approximation	Nonstationary Set
H	30	51
M	30	51
L	30	51
R1	30	51
R2	30	51
R3	30	51
H-1	45	68
-2	60	85
-3	75	102
-4	90	119
-5	--	136
-6	--	153
R1-1	45	68
-2	60	85
-3	75	102
-4	90	119
-5	--	136
-6	--	153

homogeneity tests based on moving regressions and the analysis of variance, Quandt's log-likelihood ratio, and the Chow test.

The above five procedures were applied to the sample of 197 securities for an interval of 100 observations. After three computer runs, securities were classified as having betas that were either stable, unstable, or indeterminate. A search was made of the 100 observation intervals and subintervals; securities satisfying the criteria for the two sets of portfolios were discovered.

Securities falling in these subintervals were used to form the portfolios of the two sets. For the nonstationary set, 153 securities qualified, and for the stationary approximation set, 90 securities qualified. Within the nonstationary set, 17 portfolios were formed, differing, for the most part, in either size or type. The stationary approximation set had 13 portfolios, formed in the same way as those of the nonstationary set.

CHAPTER 5

DATA ANALYSIS PROCEDURES

In order to empirically determine if diversification has eliminated prediction error, it is necessary to have measures which indicate whether or not this elimination has occurred. The purpose of this chapter is to specify the prediction error evaluation criteria that were used to accomplish the above objective. Preliminary to discussing each of the criteria, the beta predictors used in the empirical study are described. Following this description, the four evaluation criteria are presented and each is individually discussed in the order presented.

The Beta Predictors

The two prediction models mentioned in Chapter 3 are the OLS prediction model (i.e., OLS in the context of the market model) and the Bayesian prediction model. The OLS prediction model simply calculates a beta estimate using data from the prediction period and uses this estimate as the predictor for the holding period beta. The Bayesian prediction model uses a Bayesian adjustment procedure on the OLS estimator.

The Bayesian procedure was suggested by Vasichek (1973) and takes the following form:

$$\hat{\beta}_{adj} = K\ \beta_{prior} + (1-K)\ \beta_{sample}$$

where:

$$k = (1/S^2_{prior})\ /\ (1/S^2_{prior} + 1/S^2_{sample}),$$

$\hat{\beta}_{adj}$ = the Bayesian adjusted beta, which is the expected value of the posterior distribution, and is used as a predictor for the holding period beta,

β_{sample} = the beta estimated from the prediction period sample data using OLS,

β_{prior} = the expected value of the prior distribution,

S^2_{prior} = the variance of the prior distribution,

S^2_{sample} = the variance of the prediction period sample beta.

For either set of portfolios, the prior information consists of the cross-sectional distribution of all the prediction period betas. The mean of this distribution is β_{prior} and is estimated by the mean of all the prediction period sample betas. The observed variance of this distribution cannot be used as S^2_{prior} since it reflects dispersion from two sources; namely, the dispersion of the underlying betas and the dispersion introduced by the sampling error present in the observed betas (Beaver and Manegold 1975). However, using the same approach as Beaver and Manegold (1975) and Bogue (1972), an operational assessment of S^2_{prior} can be calculated. First, it is assumed that the two sources of dispersion are uncorrelated and that the variance of the cross-sectional distribution of the observed betas is the sum of the underlying beta distribution and the variance of the measurement error. An estimate of the measurement error variance is calculated by averaging the variances of the sample betas. This average variance is subtracted from the observed cross-sectional variance resulting in an assessment of S^2_{prior}, and, consequently, allows the calculation of the Bayesian adjusted beta.

As mentioned in Chapter 3, the Bayesian predictors only receive limited attention in this study. In fact, Bayesian predictors are used only for the risk-partitioned incremental portfolios of the nonstationary set. Bayesian predictors are only of interest if diversification fails to eliminate prediction error when OLS predictors are used. Since the sampling error component of prediction error can be reduced to a level of unimportance by increasing the number of observations, it was decided to forego the investigation of the Bayesian predictors for the stationary approximation set.

The Bayesian predictors were used only for the risk-partitioned incremental portfolios of the nonstationary set. They were not applied to the other portfolios of the nonstationary set for the reasons set forth below. The prior information used in the Bayesian adjustment is the cross-sectional distribution of the 153 securities of the nonstationary set. However, the portfolios, Hi-6 and R1-6 contain all 153 securities; therefore, the Bayesian adjustment will have no effect on the betas of these portfolios. If prediction error remains at this level, then the Bayesian adjustment cannot be of any assistance. Nonetheless, it is possible that the Bayesian procedure could help with some of the other portfolios of this set. However, it is unlikely that the randomly assigned portfolios would receive much help. Their portfolio betas are likely to be close to the mean of the cross-sectional distribution. If so, the Bayesian adjustment again will have no appreciable effect (the procedure adjusts the individual betas, and thus the portfolio beta, towards the mean of the cross-sectional distribution). Thus, the portfolios which could benefit from the adjustment are the risk-partitioned portfolios. Having discussed the predictors, the prediction error evaluation criteria can now be specified.

The Prediction Error Evaluation Criteria

In order to assess the effect of prediction error at the portfolio level, four evaluation criteria were used:

1. coefficient of variation,
2. absolute percentage deviation,
3. root prediction error, and
4. statistical tests of nonstationarity.

All four criteria were applied to the OLS estimators. However, only the second and third criteria were applied to the Bayesian predictors because information was not available to permit the application of the other two.

The Coefficient of Variation

The first criterion, the coefficient of variation, is compatible with the notion that the standard deviation of the sampling distribution measures the prediction error present in the estimated portfolio beta. As prediction error decreases so does the magnitude of the standard deviation. However, since the behavior of prediction error is to be examined as the number of securities in a portfolio increase, and since the portfolio beta may change as the size of the portfolio increases, it becomes impossible to compare standard deviations as the distributions of beta have different means.

To achieve comparability, the measurement of prediction error must be changed into relative forms. The most common procedure is to express the standard deviation as a percentage of the average around which deviations are taken (Chou 1972). This measure of relative variation is called the coefficient of variation and will be represented by the symbol CV. Thus, CV = $S_{\hat{\beta}_p} / \hat{\beta}_p$, where $S_{\hat{\beta}_p}$ is the standard error of $\hat{\beta}_p$.

If the portfolio beta is stationary, either because all of the component betas are stationary or because the nonstationary effects wash out, then the coefficient of variation of the prediction period beta measures the prediction error. However, if nonstationary effects persist at the portfolio level, then the variance of beta is only one component of prediction error, and, consequently, does not measure the total prediction error. For the stationary aprroximation set CV was expected to be a reasonable measure of prediction error. The reasonableness of CV as an evaluation measure for the other set depends on evidence from the other three evaluation criteria.

The Independence Issue. The variance of the portfolio beta is also expected to help clarify the error effect of nonindependence of the individual betas. If the betas are independent, then $Var(\hat{\beta}_p) = \Sigma Var(\hat{\beta}_i)/N^2$. If, however, they are not independent, $Var(\hat{\beta}_p) = \Sigma Var(\hat{\beta}_i)/N^2 + \sum_{i}\sum_{\substack{j \\ i \neq j}} Covar(\hat{\beta}_i, \hat{\beta}_j)$. To assess the impact of the covariance term if nonindependence exists, the variance of the individual sample betas was estimated using the Timvar OLS regression package. These variance estimates were then summed and divided by N^2 yielding an estimate of $Var(\hat{\beta}_p)$ if independence exists. This estimate, called $S^2_{\hat{\beta}_p} - I$, was then compared with the observed estimate of $Var(\hat{\beta}_p)$ which was calculated by regressing $\Sigma R_{it}/N$ on R_{mt} where R_{it} is the return on security i at time t and R_{mt} is the market return at time t. Any difference can be attributed to a nonzero covariance term. Also, the magnitude of the difference can be used to evaluate the seriousness of the violation of the independence assumption.

Absolute Percentage Deviation

The second evaluation criterion, the absolute percentage deviation, is identical, in concept, to the one used by Beaver et al. (1970). Prediction error is the difference between the predicted beta and the true beta of the holding period. However, the underlying beta of the holding period is never actually observed. An estimate, $\hat{\beta}_{pH}$, of the holding period beta can be calculated by regressing the portfolio return on the market return during this period. Securities were selected such that the evidence from the nonstationary tests were consistent with stationary betas for the holding period. It is then assumed that the portfolio beta estimated from the holding period is a surrogate or standard for evaluating the predictive ability of the prediction period beta. The absolute percentage deviation (APD) is defined as:

$$APD = |\hat{\beta}_{pP} - \hat{\beta}_{pH}| / \hat{\beta}_{pH} ,$$

where:

$\hat{\beta}_{pP}$ = the portfolio beta estimated from the prediction period,

$\hat{\beta}_{pH}$ = the portfolio beta estimated from the holding period.

Again, for purposes of comparability, the absolute difference is expressed as a percentage of the standard. If diversification works, then APD approaches zero as the number of securities in a portfolio increases. This result follows because under the assumption that diversification eliminates prediction error, both $\hat{\beta}_{pP}$ and $\hat{\beta}_{pH}$ approach β_{pH} as the number of securities increases, and, therefore, the numerator of APD would approach zero.

Root Prediction Error

The third criterion, root prediction error, uses the return distribution in order to develop a measure of prediction error. The relationship between the return of a portfolio in the holding period and the market return in the holding period can be expressed by the market model:

$$R_{pH} = \alpha_{pH} + \beta_{pH}R_{mH} + e_{pH}, \quad (5.1)$$

where α_{pH}, β_{pH} are the true parameters for the holding period and R_{pH} and R_{mH} are, respectively, the actual return of the portfolio in the holding period and the actual return of the market factor in the holding period. The residual, e_{pH}, reflects the imperfect correlation of R_{pH} with the market factor, R_{mH}, as would be the case for efficient portfolios lying on the curved frontier of portfolios consisting exclusively of risky assets.

If, now, predictors, $\hat{\alpha}_{pP}$ and $\hat{\beta}_{pP}$, generated from the prediction period are substituted for α_{pH} and β_{pH} in equation (5.1), the result is:

$$R_{pH} = \hat{\alpha}_{pP} + \hat{\beta}_{pP}R_{mH} + Z_p \quad (5.2)$$

The relationship between the predictors and the holding period parameters can be expressed as:

$$\hat{\alpha}_{pP} = \alpha_{pH} + U_{\alpha} , \quad (5.3)$$

$$\hat{\beta}_{pP} = \beta_{pH} + U_{\beta} , \quad (5.4)$$

where:

U_α = the prediction error on α_{pH}, and

U_β = the prediction error on β_{pH} .

Next, substituting the right hand side of (5.3) and (5.4) for $\hat{\alpha}_{pP}$ and $\hat{\beta}_{pP}$ in (5.2) yields:

$$R_{pH} = \alpha_{pH} + U_\alpha + \beta_{pH}R_{mH} + U_\beta R_{mH} + Z_p$$

and

$$R_{pH} - (\alpha_{pH} + \beta_{pH}R_{mH}) = U_\alpha + U_\beta R_{mH} + Z_p ,$$

which implies (by equation (5.1))

$$e_{pH} = U_\alpha + U_\beta R_{mH} + Z_p ,$$

whence

$$Z_p = e_{pH} - c_p$$

where

$$c_p = U_\alpha + U_\beta R_{mH} .$$

The term, c_p, is the additional effect on the residual, e_{pH} due to the presence of the prediction error, U_α and U_β.

The expected value of the squared error, z_p^2, is

$$E(Z_p^2) = E(e_{pH}-c_p)^2 = E(e_{pH}^2 - 2e_{pH}c_p + c_p^2)$$

$$= E(e_{pH}^2) - 2E(e_{pH}c_p) + E(c_p^2) ,$$

and for a given $\hat{\alpha}_{pP}$ and $\hat{\beta}_{pP}$,

$$E(Z_p^2|\hat{\alpha}_{pP},\hat{\beta}_{pP}) = Var(e_{pH}) + U_\alpha^2 + 2U_\alpha U_\beta E(R_{mH}) + U_\beta^2 E(R_{mH}^2)$$

since $E(e_{pH}c_p) = U_\alpha E(e_{pH}) + U_\beta E(e_{pH}R_{mH}) = 0$ by the market model assumptions.

The effect of U_α will be ignored because the parameter α is usually not statistically significant or of such small magnitude (usually of the order 10^{-3} or less) that computationally $U_\alpha^2 + 2U_\alpha U_\beta E(R_{mH})$ has no important impact on the empirical analysis of prediction error. Thus, a good approximation[1] of the effect of prediction error on β_{pH} can be expressed as:

$$U_\beta^2 E(R_{mH}^2) = E(Z_p^2|\hat{\alpha}_{pP},\hat{\beta}_{pP}) - Var(e_{pH}) \qquad \text{and}$$

$$U_\beta^2 = [E(Z_p^2|\hat{\alpha}_{pP},\hat{\beta}_{pP}) - Var(e_{pH})] \;/\; E(R_{mH}^2) \;,$$

$$U_\beta = \{[E(Z_p^2|\hat{\alpha}_{pP},\hat{\beta}_{pP}) - Var(e_{pH})] \;/\; E(R_{mH}^2)\}^{1/2} \qquad (5.5)$$

The expression for U_β in (5.5) is called the root prediction error (RPE). Operationally, unbiased estimates of the terms on the right hand side of (5.5) were used to obtain an estimate of RPE. To estimate $Var(e_{pH})$, the mean square residual obtained from regressing R_{pH} on R_{mH} for the observations of the holding period was used. Unbiased estimates for $E(Z_p^2|\hat{\alpha}_{pP},\hat{\beta}_{pP})$ and $E(R_{mH}^2)$ require more analysis. First, note that for a random variable, X,

$$E[X-E(X)]^2 = E(X^2) - [E(X)]^2 = Var(X)$$

which, in turn, implies

$$E(X^2) = Var(X) + [E(X)]^2 \; . \qquad (5.6)$$

Let $\bar{E}$ be an unbiased sample estimate of $E(X^2)$; then $E(\bar{E}) = E(X^2)$. Furthermore, the following expression yields an $\bar{E}$ that is an unbiased

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1. This measure differs from the conventional root mean square error (RMSE) which was used by Bogue (1972) and Rosenberg and McKibben (1973). The conventional RMSE is simply $[E(Z_p^2|\hat{\alpha}_{pP},\hat{\beta}_{pP})]^{1/2}$.

estimate of $E(X^2)$:

$$\bar{\bar{E}} = S_x^2 + \bar{X}^2 - S_x^2/N \qquad (5.7)$$

where:

S_x^2 = the sample variance divided by the appropriate degrees of freedom to make it an unbiased estimate of Var(X),

X = the sample mean,

N = the sample size.

It can be shown that $\bar{\bar{E}}$ is an unbiased estimate by the following analysis: from equation (5.7),

$$E(\bar{\bar{E}}) = E(S_x^2) + E(\bar{X}^2) - E(S_x^2)/N,$$

and, using the relationship of equation (5.6) for $E(\bar{X}^2)$,

$$E(\bar{\bar{E}}) = E(S_x^2) + [\mathrm{Var}(\bar{X}) + [E(\bar{X})]^2] - E(S_x^2) / N$$

$$= \mathrm{Var}(X) + \mathrm{Var}(X)/N + [E(X)]^2 - \mathrm{Var}(X)/N$$

$$= \mathrm{Var}(X) + [E(X)]^2$$

which, by equation (5.6),

$$= E(X^2).$$

Thus, the estimate of $E(Z_p^2|\hat{\alpha}_{pP},\hat{\beta}_{pP})$ and $E(R_{mH}^2)$ were obtained by substituting the respective sample variances and sample means, calculated from the holding period observations, into equation (5.7). And, when these estimates (with the estimate of $\mathrm{Var}(e_{pH})$) are substituted into equation (5.5), a sample estimate of RPE, $\hat{RPE}$, is the result. An example for a small, arbitrary, discrete population is given in Appendix E.

Statistical Tests of Nonstationarity

The last criterion, the statistical tests of nonstationarity, is used to determine the absence or presence (at the portfolio level) of the nonstationary component of prediction error. The TIMVAR tests were run on the portfolio beta for the prediction period and the holding period. In addition, the Chow test was run to determine if the portfolio beta of the prediction period was significantly different from the portfolio beta of the holding period. Statistical significance of these tests is taken as evidence that the nonstationary error component persists at the portfolio level (i.e., statistical significance is evidence that diversification was not successful in eliminating prediction error). Nonsignificance of the tests is interpreted as evidence consistent with the elimination of the nonstationary error component.

Summary

The goal of this chapter was to present the procedures that are used in the analysis of the empirical results. The first section discussed the beta predictors. There are two types of predictors used in the study -- OLS predictors and Bayesian predictors. The Bayesian predictors are applied only to the risk-partitioned incremental portfolios of the nonstationary set, whereas the OLS predictors are applied to all portfolios of both sets.

Next, the prediction error evaluation criteria are identified and explained. The four criteria that are discussed in the chapter are the coefficient of variation, absolute percentage deviation, root prediction error, and statistical tests of nonstationarity. The middle two criteria are applied to the Bayesian predictors and all four criteria are applied to the OLS predictors.

The coefficient of variation is a measure of the relative dispersion of the sampling distributions of the portfolio betas and therefore measures prediction error (if nonstationary effects persist at the portfolio level it is an incomplete measure). The absolute percentage deviation was defined as the absolute difference between the estimated portfolio betas of the prediction and holding periods, divided by the estimated holding period portfolio beta. The root prediction error is a point estimate of the actual prediction error on the portfolio beta of the holding period. And, finally, the statistical tests of nonstationarity, applied at the portfolio level, give indications of the absence or presence of the nonstationary component of prediction error.

CHAPTER 6

ANALYSIS OF THE RESULTS

This chapter presents an analysis of the results of the empirical study of prediction error. The first section deals with the results pertaining to the stationary approximation set. Within this section the outcomes for each of the four prediction error evaluation criteria are presented for the OLS predictors. In order to determine if the stationary approximation set is mostly free from the nonstationary error component of prediction error, the fourth criterion (i.e., the statistical tests of nonstationarity) is discussed first. Discussion of the coefficient of variation, APD and $R\hat{P}E$ then follows. The issue of independence is also discussed in the first section.

The results pertaining to the nonstationary set are given in the second section. Again, beginning with the statistical tests of nonstationarity, the outcomes for all four evaluation criteria are presented. In addition, the results for the Bayesian predictors are given for the two relevant evaluation criteria. The final section of the chapter contains a comparative discussion of the results for the two sets of portfolios.

Results -- Stationary Approximation Set

Statistical Tests of Nonstationarity

The results of the application of the TIMVAR and Chow tests are summarized in Tables 6.1, 6.2, and 6.3. Table 6.1 is concerned with the six portfolios of size 30, and Tables 6.2 and 6.3 deal with portfolios increasing in size by increments of 15, with high risk and random bases, respectively. For the most part, the results of the statistical tests were consistent with stationarity.

From Table 6.1, it can be seen that the TIMVAR tests were applied to the betas of the portfolios in both the prediction and holding periods. There was only one TIMVAR test in one time period that gave a significant result. Specifically, the homogeneity test for a moving regression of length four was significant at the .05 level for the holding period of the R2 portfolio.

The Chow test was also run on the six portfolios listed in Table 6.1. A significant difference between the portfolio betas of the prediction period and holding period was indicated for the high risk portfolio. For purposes of further investigation, TIMVAR was run on the entire 30 observations of the two periods of this high risk portfolio. The homogeneity test for a moving regression of length 15 rejected the null hypothesis of stationarity of the regression coefficients at a significance level of .05, which, for this length is identical to the Chow test. Quandt's log-likelihood ratio test indicated that the point of disturbance occurred at observation 15.

Table 6.1. Nonstationary Tests for Individual Portfolios, Stationary Approximation Set.

	H		M		L		R1		R2		R3	
	P	H	P	H	P	H	P	H	P	H	P	H
Cusum BW	.30	.67	.32	.44	.22	.64	.22	.42	.43	.85	.23	.62
Cusum Sq.BW	.21	.20	.20	.19	.21	.33	.33	.30	.14	.22	.18	.21
Cusum FW	.20	.44	.62	.36	.59	.38	.69	.33	.50	.41	.30	.49
Cusum Sq.BW	.19	.18	.21	.15	.33	.43	.44	.35	.14	.16	.21	.20
MR1 4	.58	2.4	.61	1.1	.32	.38	.45	.71	.86	5.7 [a]	1.0	1.5
MR2 6	.40	2.7	.84	.48	.44	.32	.97	.43	.26	2.9	.38	.91
Chow	3.5 [a]		1.1		2.7		1.6		1.8		1.9	
N	30		30		30		30		30		30	

a. Significant at the .05 level.

Table 6.2. Nonstationary Tests for Increasing Portfolio Size, Stationary Approximation Set, High Risk Base.

	H		1		2		3		4	
	P	H	P	H	P	H	P	H	P	H
Cusum BW	.30	.67	.23	.57	.26	.55	.26	.62	.19	.64
Cusum Sq.BW	.21	.20	.24	.17	.22	.17	.24	.12	.27	.18
Cusum FW	.20	.44	.33	.38	.45	.41	.60	.44	.55	.45
Cusum Sq.FW	.19	.18	.21	.10	.19	.08	.29	.19	.32	.22
MR1 4	.58	2.4	1.0	2.2	1.2	2.2	.69	1.9	.79	1.6
MR2 6	.40	2.7	.27	.75	.48	.98	.34	.74	.49	.91
Chow	3.5[a]		2.26		2.25		2.39		2.12	
N	30		45		60		75		90	

a. Significant at the .05 level.

Table 6.3. Nonstationary Tests for Increasing Portfolio Size, Stationary Approximation, Random Base.

	R1		1		2		3		4	
	P	H	P	H	P	H	P	H	P	H
Cusum BW	.22	.42	.22	.49	.21	.59	.19	.63	.18	.62
Cusum Sq.BW	.33	.30	.35	.22	.34	.12	.29	.15	.28	.14
Cusum FW	.69	.33	.73	.37	.76	.38	.71	.45	.63	.44
Cusum Sq.FW	.44	.35	.37	.23	.43[a]	.19	.39	.14	.34	.18
MR1 4	.45	.71	.45	1.1	.42	1.8	.45	1.9	.67	1.9
MR2 6	.97	.43	.84	.54	.46	.83	.35	1.1	.51	.94
Chow	1.61		1.75		2.69		2.27		2.47	
N	30		45		60		75		90	

a. Significant at the .05 level.

However, as Table 6.2 demonstrates, there are no nonstationary indications for any of the portfolios formed by adding increments of 15 to the high risk portfolio. That is, none of the TIMVAR tests or the Chow tests for the portfolios H-1, H-2, H-3, and H-4 is significant at a level of .05. Apparently, whatever nonstationary effects are present in the high risk portfolio diminish as securities from the other uncontaminated portfolios are added.

Only one of the TIMVAR and Chow tests is significant for the portfolios of Table 6.3. The cusum of squares test based on forward recursion is significant for the prediction period of the R1-2 portfolio. Overall, for the portfolios listed in Tables 6.1 to 6.3, 26 time periods were subjected to the six TIMVAR tests for a total application of 156.[1] There were only two of the 156 applications of the TIMVAR tests that gave significant results. There were a total of 13 Chow tests (six from Table 6.1, four from Table 6.2 and three from Table 6.3). Of the 13 Chow tests, only one was significant. Thus, it appears that the nonstationary component of prediction error is not of much importance for the stationary approximation set. This conclusion seems even more reasonable when one considers that the number of nonstationary indications does not exceed the expected number for a .05 level of significance.

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1. From Table 6.1, 6 tests x 2 periods x 6 portfolios = 72; from Table 6.2, 6 tests x 2 periods x 4 portfolios = 48; and, from Table 6.3, 6 tests x 2 periods x 3 portfolios = 36 (since R1-4 is the same as Hi-4, it is not counted again). Thus, 72 + 48 + 36 = 156.

Coefficient of Variation

Given approximate stationarity, the coefficient of variation becomes a measure of most of the prediction error in the OLS predictors (i.e., sampling error). Tables 6.4, 6.5 and 6.6 (appearing on the last three pages of the section discussing the stationary approximation set) summarize the results of the application of this measure to the portfolios of the stationary approximation set. These tables also give the results of the remaining two evaluation criteria and other pertinent information, all of which is discussed later in this chapter.

From Table 6.4, CV_P (P indicates the prediction period value) for the six portfolios of size 30, ranges from .097 to .212. The average of CV_P is .143. Clearly, diversification has not eliminated prediction error, at least for portfolios of size 30. Some idea of the importance of the remaining prediction error can be obtained by constructing confidence intervals around the predictors. For example, the predictor of the holding period beta of the medium risk portfolio is 1.35 (see Table 6.4). An investor, using this predictor, would be 95 percent confident that 1.35 is within 32.4 percent of the true beta.[2] The width of the confidence interval is not impressive.

It is possible that portfolios that contain 30 securities are not large enough to effectively eliminate prediction error. Tables 6.5 and 6.6 present results for an increasing portfolio size. From Table 6.5,

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2. The confidence interval is constructed in the usual way, i.e., $\beta \pm t_{1-\alpha/2} \cdot S_{\hat{\beta}_p} / \hat{\beta}_p$. Of course, it is assumed that the betas of the prediction and holding periods are equal. The results of the non-stationary tests were essentially consistent with this assumption.

Table 6.4. Prediction Error Measures for Individual Portfolios, Stationary Approximation Set.

	H	M	L	R1	R2	R3
$\hat{\beta}_{pP}$	2.09	1.35	.88	1.36	1.53	1.38
$S_{\hat{\beta}_{pP}}$	.202	.202	.186	.187	.196	.181
$S_{\hat{\beta}_{pP}}$-I	.153	.118	.112	.118	.128	.119
CV_P	.097	.150	.212	.137	.128	.131
CV_P-I	.073	.088	.128	.087	.084	.086
APD	.462	.121	.109	.172	.122	.247
$R\hat{P}E$	.65	.31	.49	.30	.38	.31
$\hat{\beta}_{pH}$	1.43	1.20	.99	1.16	1.36	1.11

Table 6.5. Prediction Error Measures for Increasing Portfolio Size, Stationary Approximation Set, High Risk Base.

	H	1	2	3	4
$\hat{\beta}_{pP}$	2.09	1.88	1.70	1.55	1.42
$S_{\hat{\beta}_{pP}}$	.202	.174	.168	.154	.153
$S_{\hat{\beta}_{pP}}$-I	.153	.116	.097	.083	.075
CV_P	.097	.093	.099	.099	.107
CV_P-I	.073	.062	.057	.053	.053
APD	.462	.341	.284	.248	.173
$R\hat{P}E$	.65	.44	.38	.32	.29
$\hat{\beta}_{pH}$	1.43	1.40	1.33	1.24	1.21
N	30	45	60	75	90

Table 6.6. Prediction Error Measures for Increasing Portfolio Size, Stationary Approximation Set, Random Base.

	R1	1	2	3	4
$\hat{\beta}_{pP}$	1.36	1.40	1.38	1.39	1.42
$S_{\hat{\beta}_{pP}}$	.187	.197	.170	.162	.153
$S_{\hat{\beta}_{pP}}$-I	.118	.098	.085	.076	.075
CV_P	.137	.140	.123	.117	.107
CV_P-I	.087	.070	.061	.054	.053
APD	.172	.137	.124	.158	.173
$R\hat{P}E$	.30	.34	.36	.31	.29
$\hat{\beta}_{pH}$	1.16	1.24	1.23	1.20	1.21
N	30	45	60	75	90

CV_p increases from a value of .097 for the 30 security portfolio to a value of .107 for the 90 security portfolio. And, in Table 6.6, CV_p decreases from .137 for the 30 security portfolio to .107 for the 90 security portfolio. Thus, for the portfolios of decreasing risk of Table 6.5, increasing the number of securities has little impact on CV_p. For the random portfolios of Table 6.6, increasing the portfolio size did reduce CV_p somewhat.

However, the value of CV_p for the 90 security portfolio is still .107. The predictor for this portfolio is 1.42. Again, constructing a confidence interval around the predictor gives an impression of the importance of the prediction error remaining at this level. An investor could view 1.42 as being within 23.1 percent of the true beta with 95 percent confidence. The results of the CV_p analysis indicates that diversification has not reduced prediction error to a level of unimportance.

APD and $R\hat{P}E$

Whereas CV_p is a measure of prediction error that can occur, APD and $R\hat{P}E$ are measures of prediction error that did occur.[3] For the portfolios of size 30 in Table 6.4, APD ranges from .109 to .462 with an average of .205 and $R\hat{P}E$ ranges from .30 to .65 with an average of .4. Again, consistent with the CV_p results, diversification has not eliminated prediction error for the portfolios with 30 securities.

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3. Since $\hat{\beta}_i$ is a random variable, so is the prediction error, $U_i = \hat{\beta}_i - \beta_i$. CV_p is a measure of the <u>expected</u> squared error. APD and RPE are measures of <u>particular</u> values of the random variable, U_i.

Both APD and $R\hat{P}E$ display a similar pattern with respect to the effect an increasing portfolio size has on prediction error. Table 6.5 shows prediction error decreasing from .462 to .173 and from .65 to .29 for APD and $R\hat{P}E$, respectively, when the high risk portfolio is used as the base. Table 6.6 shows prediction error fluctuating, but changing little, when R1 is used as the base. That is, APD and $R\hat{P}E$ are, respectively, .172 and .30 for the 30 security portfolio and .173 and .29 for the 90 security portfolio. Diversification seems to benefit the portfolios of Table 6.5 but does virtually nothing for those of Table 6.6. One possible explanation of this behavior is given in the subsection following this one.

First, however, some final comments need to be made. For the largest portfolio (i.e., the 90 security portfolio) the statistical tests were consistent with the absence of any nonstationary effects. However, with respect to the sampling error component, the values of CV_p, APD, and $R\hat{P}E$ were, respectively, .107, .173, and .29. Thus, it appears that for the stationary approximation set, diversification has not been successful in eliminating prediction error.

Independence Assumption

A possible reason for the persistence of prediction error, even for a large portfolio could be nonindependence of the individual betas. If this nonindependence exists, then as the number of securities increases, $Var(\hat{\beta}_p)$ will approach the average covariance among the betas of the securities constituting the portfolio. An estimate of what the variance of $\hat{\beta}_p$ would be if the betas are independent can be calculated by summing the sample variances of the individual estimated betas and dividing by N^2. The square root of this estimate is $S_{\hat{\beta}_{pP}}-I$ and is presented in Tables 6.4, 6.5, and 6.6 for the various portfolios.

An estimate of the average covariance among the betas of the various portfolios can be calculated by noting that:

$$S^2_{\hat{\beta}_{pP}} = \sum_{i=1}^{N} S^2_{\hat{\beta}_{iP}}/N^2 + [(N-1)/N]\,\overline{\widehat{Covar}} \quad (6.1)$$

and

$$S^2_{\hat{\beta}_{pP}}-I = \sum_{i=1}^{N} S^2_{\hat{\beta}_{iP}}/N^2$$

where

$\overline{\widehat{Covar}}$ = the estimated average covariance of the betas in a given portfolio,

and

$S^2_{\hat{\beta}_{pP}}-I$ = the estimate of the portfolio variance given independence of the component betas.

Solving equation (6.1) for $\overline{\widehat{Covar}}$ yields the desired result:

$$\overline{\widehat{Covar}} = [N/(N-1)][S^2_{\hat{\beta}_{pP}} - S^2_{\hat{\beta}_{pP}}-I] \quad (6.2)$$

From equation (6.1), it can be seen that the estimate of prediction error, $S^2_{\hat{\beta}_{pP}}$, can be broken into the sum of two component estimates -- $S^2_{\hat{\beta}_{pP}}-I$ and $\overline{\widehat{Covar}}$. If nonindependence exists, then $\overline{\widehat{Covar}}$ will become the major component of $S^2_{\hat{\beta}_{pP}}$ as the number of securities increases since the variance of the estimated portfolio beta approaches the average covariance (see the nonindependence section of Chapter 3). In other words, if $\overline{\widehat{Covar}}$ is expressed as a percentage of $S^2_{\hat{\beta}_{pP}}$, one would expect this percentage to increase as the size of the portfolio increases if nonindependence exists.

Using equation (6.2) and the data from Tables 6.4 to 6.6, it is possible to calculate $\overline{\widehat{Covar}}$ for each portfolio and then express it as a percentage of $S^2_{\hat{\beta}_{pP}}$. For the portfolios of size 30 in Table 6.4, $\overline{\widehat{Covar}}$ ranges from 44.1 percent to 68.1 percent. Of real interest is the behavior of $\overline{\widehat{Covar}}$ (as a percentage of $S^2_{\hat{\beta}_{pP}}$) as the portfolio size increases. When these percentages are calculated from Table 6.5, they increase (strictly) from 44.1 percent for the high risk base portfolio to 76.8 percent for the H-4 portfolio. For the random portfolios of Table 6.6, the percentage increases from 62.3 for the R1 portfolio to 76.8 for the R1-4 portfolio. However, in this case all the incremental portfolios have percentages around 77 percent. The impression received from this analysis is that prediction error is approaching some average covariance value. That is, it seems that nonindependence is affecting the ability of diversification to eliminate prediction error.

With the evidence of nonindependence of betas, a possible explanation exists for the pattern displayed by APD and $\widehat{RPE}$ for an increasing portfolio size. It is possible that prediction error decreases with increasing portfolio size for the high risk base because the nonstationary error effects (i.e., $\overline{V}^2/N + \overline{V_i V_j}/N$) seemingly present in the high risk portfolio are being diluted. Then, when the R1 base is used, little change occurs because the benefits of diversification are essentially exhausted (the nonstationary effects are randomly scattered through R1, R2, and R3) and the remaining prediction error is largely due to covariance effects.

Another way to decrease the sampling error component of prediction error is to increase the number of observations used in the prediction period. However, this process increases the likelihood of encountering structural changes in individual betas, which, in turn, introduces the nonstationary error component into the portfolio. The forthcoming discussion is concerned with the analysis of the empirical results that relate to diversification and its effect on the nonstationary error component.

Results -- Nonstationary Set

Statistical Tests of Nonstationarity

The results of the application of the TIMVAR and Chow tests are summarized in Tables 6.7, 6.8, and 6.9. The empirical design was intended to produce holding period portfolio betas that were free from nonstationary effects. Examination of the results of the statistical tests for the holding periods (for all three tables) reveals that all of the 17 portfolios have betas that are consistent with stability.

For the prediction period, securities were selected so that a substantial portion had evidence of nonstationary betas. The last row of Tables 6.7, 6.8, and 6.9 indicates the minimum percentage of securities that have been identified as having unstable betas within the prediction period. For the 17 portfolios listed in the three tables, the range of the minimum percentage is from 19.6 percent to 49 percent. If these known nonstationary effects are eliminated by diversification then the portfolio beta will be stable over time. Significant results of the TIMVAR tests would indicate that the nonstationary error component can

Table 6.7. Nonstationary Tests for Individual Portfolios, Nonstationary Set.

	Hi		M		L		R1		R2		R3	
	P	H	P	H	P	H	P	H	P	H	P	H
Cusum BW	.84	.45	.96[a]	.61	.41	.27	.75	.49	.86	.49	.71	.33
Cusum Sq.BW	.23[a]	.24	.20[a]	.25	.18	.32	.28[a]	.22	.18	.17	.30[a]	.22
Cusum FW	.81	.52	.47	.53	.34	.42	.72	.46	.49	.64	.73	.36
Cusum Sq.FW	.22[a]	.26	.21[a]	.22	.22[a]	.23	.27[a]	.32	.23[a]	.24	.30[a]	.28
MR1	1.2	1.3	.79	1.0	.52	.24	1.0	.72	.66	.62	.97	1.1
MR2	.71	1.9	.60	.73	.77	.38	.89	1.1	.48	.79	.61	1.1
Chow	11.2[a]		3.51[a]		.206		3.2[a]		4.45[a]		6.73[a]	
N	51		51		51		51		51		51	
Min. Percent	49		27.5		19.6		29.4		27.5		39.2	

a. Significant at a level of .05.

Table 6.8. Nonstationary Tests for Increasing Portfolio Size, Nonstationary Set, High Risk Base.

	Hi		1		2		3		4		5		6	
	P	H	P	H	P	H	P	H	P	H	P	H	P	H
Cusum BW	.84	.45	.83	.47	.93	.48	.93	.49	.87	.46	.84	.45	.80	.44
Cusum Sq.BW	.23[a]	.24	.23[a]	.24	.26[a]	.22	.27[a]	.20	.28[a]	.21	.28[a]	.22	.28[a]	.22
Cusum FW	.81	.52	.75	.53	.75	.54	.71	.54	.74	.50	.69	.48	.65	.49
Cusum Sq.FW	.22[a]	.26	.23[a]	.25	.26[a]	.23	.27[a]	.23	.28[a]	.27	.28[a]	.28	.28[a]	.30
MR1	1.2	1.3	1.0	1.2	1.0	1.2	.99	1.2	.91	.97	.88	.86	.79	.81
MR2	.71	1.9	.67	1.6	.55	1.5	.58	1.4	.55	1.2	.52	1.1	.51	1.1
Chow	11.2[a]		10.8[a]		5.09[a]		8.7[a]		7.87[a]		6.73[a]		5.54[a]	
N	51		68		85		102		119		136		153	
Min. Percent	49		41.1		38.8		38.2		36.1		34.6		32.7	

a. Significant at a level of .05.

Table 6.9. Nonstationary Tests for Increasing Portfolio Size, Nonstationary Set, Random Base.

	R1		1		2		3		4		5		6	
	P	H	P	H	P	H	P	H	P	H	P	H	P	H
Cusum BW	.75	.49	.77	.47	.79	.47	.83	.49	.75	.44	.77	.46	.81	.44
Cusum Sq.BW	.28[a]	.22	.22[a]	.21	.25[a]	.21	.25[a]	.20	.25[a]	.23	.26[a]	.22	.28[a]	.22
Cusum FW	.72	.46	.67	.50	.65	.54	.63	.55	.64	.50	.68	.51	.68	.49
Cusum Sq.FW	.27[a]	.32	.25[a]	.31	.26[a]	.29	.27[a]	.29	.26[a]	.26	.27[a]	.30	.27[a]	.30
MR1	1.0	.72	1.1	.61	.91	.61	.84	.67	.76	.65	.82	.68	.88	.80
MR2	.89	1.1	1.1	.90	.81	.82	.65	.95	.65	.82	.73	.95	.63	1.1
Chow	3.2[a]		3.55[a]		4.92[a]		4.44[a]		5.22[a]		5.46[a]		5.54[a]	
N	51		68		85		102		119		136		153	
Min. Percent	31.4		33.8		31.8		29.4		32.8		33.8		32.7	

a. Significant at a level of .05.

be felt at the portfolio level, and, therefore, was not successfully eliminated by diversification. Moreover, a significant result for the Chow test would show there is a significant difference between the portfolio beta of the prediction period and the holding period portfolio beta. This result, of course, would be due to the nonstationary error component, which, again, would demonstrate that diversification does not work.

For the six portfolios of size 51 in Table 6.7, there were 36 applications of the TIMVAR tests in the six prediction periods. Of the 36 applications, 11 showed statistical significance at a level of .05. Each of the six portfolio betas had nonstationary indications within the prediction period. Also, of the six Chow tests, five indicated that the betas of the two periods were significantly different. Thus, diversification has not eliminated the nonstationary component of prediction error for the portfolios with 51 securities.

This same conclusion also follows for the larger portfolios of Tables 6.8 and 6.9. Each of the 11 incremental portfolios in these two tables gives indications of nonstationary betas. For these 11 portfolios there were 66 applications of the TIMVAR tests and 11 Chow tests. Of the 66 TIMVAR applications, there were 22 which displayed significance at a level of .05. All 11 Chow tests were significant.

Since the cusum of squares test is virtually the only TIMVAR test showing significance, the possibility again exists that significance is due primarily to a change in residual variance and not to a local change in the portfolio beta. The plots of the residual variance of the moving regressions were examined. Most gave the appearance of reasonable stability. However, in order to provide firmer evidence than visual observation, the Hartley test for equality of variances (Neter and Wasserman 1974, pp. 512-513) was run using the eight nonoverlapping estimates of the variance available from the moving regression of length ten. At a significance level of .05, all of the 17 portfolios in the prediction period had variances consistent with stability. Thus, the evidence indicates that nonstationary effects persist at the portfolio level.

Coefficient of Variation

Since the coefficient of variation measures only the sampling error component of prediction error, it is not a complete measure of prediction error for the nonstationary set if nonstationary effects remain at the portfolio level. Given the results of the statistical tests of nonstationarity, it appears that the function of the coefficient of variation for the nonstationary set is to indicate the amount of sampling error. Tables 6.10, 6.11, and 6.12 summarize the application of this measure for the portfolios of the nonstationary set. These tables also give the results of the remaining two evaluation criteria. Table 6.11 gives the results of APD and $\widehat{RPE}$ as applied to the Bayesian adjusted (Badj) predictors.

From Table 6.10, CV_p for the six portfolios with 51 securities ranges from .049 to .072. The average CV_p is .055. For the larger portfolios of Tables 6.11 and 6.12, CV_p ranges, respectively, from .048 to .054 and .048 to .051. The respective averages are .05 and .049.

Table 6.10. Prediction Error Measures for Individual Portfolios, Nonstationary Set.

	H	M	L	R1	R2	R3
$\hat{\beta}_{pP}$	1.90	1.19	0.747	1.230	1.30	1.33
$S_{\hat{\beta}_{pP}}$	.102	.061	.054	.062	.069	.065
$S_{\hat{\beta}_{pP}}$-I	.038	.029	.041	.045	.032	.031
CV_P	.054	.051	.072	.051	.053	.049
CV_P-I	.020	.024	.055	.037	.025	.023
APD	.721	.287	.005	.349	.354	.404
$R\hat{P}E$	.8	.24	.06	.28	.32	.38
N	51	51	51	51	51	51
$\hat{\beta}_{pH}$	1.10	.927	.751	.91	.958	.946

Table 6.11. Prediction Error Measures for Increasing Portfolio Size, Nonstationary Set, High Risk Base.

	H	1	2	3	4	5	6
$\hat{\beta}_{pP}$	1.90	1.76	1.64	1.56	1.47	1.38	1.28
$S_{\hat{\beta}_{pP}}$	.102	.091	.082	.077	.071	.065	.062
$S_{\hat{\beta}_{pP}}$-I	.038	.032	.027	.024	.022	.020	.019
CV_P	.054	.052	.050	.049	.048	.047	.048
CV_P-I	.020	.018	.017	.016	.015	.014	.015
APD	.721	.656	.577	.525	.480	.419	.376
$\hat{RPE}$	.80	.70	.58	.52	.46	.38	.29
APD Badj	.613	.531	.486	.421	.399	.319	.376
$\hat{RPE}$ Badj	.64	.55	.46	.41	.37	.33	.29
$\hat{\beta}_{pH}$	1.10	1.06	1.04	1.02	.993	.97	.93
N	51	68	85	102	119	136	153

Table 6.12. Prediction Error Measures for Increasing Portfolio Size, Nonstationary Set, Random Base.

	R1	1	2	3	4	5	6
$\hat{\beta}_{pP}$	1.23	1.24	1.25	1.26	1.28	1.27	1.28
$S_{\hat{\beta}_{pP}}$	.062	.061	.061	.062	.063	.061	.062
$S_{\hat{\beta}_{pP}}-I$	.045	.036	.031	.028	.025	.023	.021
CV_P	.051	.049	.048	.049	.049	.048	.048
CV_P-I	.037	.029	.025	.022	.020	.018	.016
APD	.349	.318	.375	.352	.376	.374	.365
$R\hat{P}E$	.28	.26	.36	.32	.34	.28	.29
$\hat{\beta}_{pH}$	.911	.939	.911	.934	.930	.923	.930
N	51	68	85	102	119	136	153

If the nonstationary effects had canceled out, then an investor could have been 95 percent confident of being within about 10 percent of the true beta. This result indicates that the sampling error component of prediction error is not nearly as important for the nonstationary set as it was for the stationary approximation set. Of course, one would expect this conclusion as 86 observations were used to estimate beta.

As a matter of interest it should be noted that increasing the number of securities in the base portfolios has little impact on CV_p. From Table 6.11, CV_p decreases from .054 for a portfolio with 51 securities to .048 for a portfolio with 153 securities; and for the portfolios of the random base (Table 6.12) CV_p decreases from .051 to .048. Apparently, most of the benefits of diversification, with respect to sampling error, have been achieved.

Again the likely cause of the inability of diversification to further reduce sampling error can be attributed to the violation of the independence assumption. That is, sampling error, as measured by $S^2_{\hat{\beta}_{pP}}$ is approaching the average covariance among the estimated betas. For example, the estimated average covariance (calculated as outlined in the previous section), expressed as a percentage of $S^2_{\hat{\beta}_{pP}}$, for the 153 security portfolio is 91.2 percent. Apparently, nearly all of the remaining sampling error component is due to covariance effects.

APD and $\hat{RPE}$

APD and $\hat{RPE}$, unlike CV_p, measure total prediction error for the nonstationary set. For the six portfolios with 51 securities in Table 6.10, APD ranges from .005 to .721, with an average of .353. $\hat{RPE}$ ranges from .06 to .8 with an average of .347. Diversification has not eliminated prediction error for these portfolios (with the possible exception of the low risk portfolio). This result, as would be expected, is consistent with the results of the statistical tests of nonstationarity.

One would expect this same kind of consistency for the portfolios of increasing size. In fact, upon examination of Tables 6.11 and 6.12, this expectation is confirmed. Table 6.11 shows APD and $R\hat{P}E$ decreasing, respectively, from .721 to .376 and .8 to .29 when the high risk base is used. Table 6.12 shows APD and $R\hat{P}E$ fluctuating, but changing little (.349 to .376 and .28 to .29, respectively) when R1 is used as the base portfolio. Diversification seems to be of some benefit for the portfolios of Table 6.11 and does nothing for those of Table 6.12.

The explanation seems apparent. The high risk base has the highest proportion of securities with nonstationary betas. Adding securities from the medium and low risk portfolios dilutes the nonstationary effects. That is, the proportion of securities with nonstationary betas decreases, with the result that the nonstationary effects are reduced by the presence of a greater number of securities with stable betas. However, for the random portfolios these effects are already spread randomly among the securities; hence no additional dilutive effect is seen as securities from R2 and R3 are randomly added to R1 (observe in Table 6.9 that the proportion of securities with nonstationary betas for R1 and each of the incremental portfolios stays about the same). The key to reducing prediction error due to nonstationary effects does not seem to be related to simply adding more securities, but rather increasing the number of securities with <u>stable</u> betas.

Thus, the overall impression is that diversification is unable to eliminate prediction error for the OLS predictors of the nonstationary set. Moreover, since sampling error has been reduced considerably by increasing the observations used in the prediction model, the major component of prediction error remaining at the portfolio level is due to nonstationary effects.

Bayesian Results. Table 6.11 also presents the results of the Bayesian adjusted predictors. There is improvement at each level with prediction error at the final level remaining essentially the same, as would be expected. For example, for the high risk base, APD is .721 whereas APD Badj is .613 and $\hat{R}PE$ is .8 whereas $\hat{R}PE$ Badj is .64. For most levels there is a 10 to 20 percent reduction in prediction error. However, even though the Bayesian adjustment procedure improved the predictive ability, it does not succeed in eliminating prediction error for any of the levels examined.

Comparison of the Two Sets

The coefficient of variation of the nonstationary set indicates that the sampling variability component of prediction error is considerably less than that of the first set. Of course this is expected as 86 observations were used to estimate the betas of the nonstationary set whereas only 15 observations were used to estimate the betas of the stationary approximation set. It seems evident that if the nonstationary effects were to wash out then the nonstationary set would provide better predictors than the stationary approximation set. For example, of the six portfolios of Table 6.10, the low risk portfolio appears to be the most nearly stable. Interestingly, it has the lowest number of securities with identified nonstationary betas within the prediction period. The coefficient of variation of this portfolio is probably a reasonable measure of prediction error. If it is a reasonable measure then one could be 95 percent confident that the estimate, .747, is within 14 percent of the true beta. The other two measures indicate a small amount of prediction error. The APD is only .005 and $\hat{RPE}$ is but .06.

Unfortunately, the other portfolios in the nonstationary set appear to have much stronger nonstationary effects. Comparison of the average values of APD and $\hat{RPE}$ from Tables 6.4 and 6.10 indicates that the nonstationary set did not predict the observed beta of the holding period as well as the stationary approximation set (i.e., .353 vs. .205) but that the average $\hat{RPE}$ was less for the nonstationary set (.347 vs. .4). Of real interest is the comparison for each of the largest portfolios. For the stationary approximation set, the 90 security portfolio has an APD and $\hat{RPE}$ of .173 and .29, respectively. This compares with the 153 security portfolio's APD and $\hat{RPE}$ of .376 and .29, respectively. Even though the nonstationary set uses many more observations to generate the predictor of the holding period beta (and thus has much less sampling error) and also has portfolios of much larger size, the success in eliminating prediction error when compared to the first set is about the same. Considering the design of the two sets and the results of the statistical tests of nonstationarity, the inability of the nonstationary set to eliminate prediction error can be largely attributed to the persistence of nonstationary effects at the portfolio level.

Summary

The objective of this chapter was to give an analysis of the results of the empirical study of prediction error. The chapter had three major sections. The first section gave an analysis of the results for the stationary approximation set. The statistical tests of nonstationarity were largely consistent with stationary portfolio betas. This result indicated that the major source of prediction error for this set was sampling error. The other three criteria, CV_p, APD, and $\widehat{RPE}$ indicated that prediction error was not eliminated for any of the portfolios of the stationary approximation set. Evidence was presented that nonindependence of the individual betas may explain why diversification did not work for this set.

The second section dealt with the nonstationary set. The statistical tests of nonstationarity indicated the presence of nonstationary effects at the portfolio level. The coefficient of variation was used to show that sampling error was reduced considerably by increasing the number of observations used in calculating the predictor. The remaining two measures, APD and $\widehat{RPE}$, both present evidence that diversification did not eliminate prediction error for the nonstationary set. Thus, since the major source of prediction error for this set is nonstationary effects, these effects evidently did not disappear at the portfolio level.

The last section entailed a brief comparison of the two sets. In comparing the two sets, it was noted that the nonstationary set would produce superior estimates if nonstationary effects were not present at the portfolio level. However, even though the nonstationary set had more securities in its portfolios, the prediction error remaining for both sets was about the same. The results of the empirical study are clear. Prediction error was not eliminated by diversification.

CHAPTER 7

CONCLUSION

The objective of this study was presented in Chapter 1, the introductory chapter. This chapter gave a brief review of capital market theory, which was considered preliminary background material for the statement of the problem being investigated by this study. According to capital market theory, an investor needs to assess two parameters, namely, the risk and expected return of his portfolio. Thus, if accounting information is to be of value to an individual investor in selecting his portfolio then it must assist him in assessing the risk-return characteristics of various portfolios. This study was restricted to the investigation of risk assessment. Capital market theory also indicates that the only variable that determines differential riskiness among securities is the systematic risk. Systematic risk of a security is defined as the security's contribution to the overall portfolio risk. Therefore, prediction of the systematic risk becomes a crucial part of portfolio analysis.

There is evidence that accounting information can improve such risk predictions, and, therefore, might be of value to individual investors. However, if the errors of the risk predictions on the individual betas cancel out in the process of portfolio formation (i.e., diversify out), then accounting information may be of no value to the individual investor in assessing the portfolio risk. The objective of this study was to determine the effect diversification has on prediction error, and, consequently, provide evidence as to whether accounting information might be of value to an individual investor in estimating the risk of his portfolio.

In order to provide a firm foundation for the analysis of the prediction error problem, the principle of diversification was discussed in Chapter 2. Diversification was defined within the context of two models -- the original Markowitz model and the simplified version of the Markowitz model called the market model. In both cases diversification was essentially a dispersion reducing activity. That is, increasing the number of securities in a portfolio essentially eliminated any portfolio dispersion caused by the dispersion of individual return distributions. With this principle in mind, the analysis of the prediction error problem was possible.

Summary of the Findings of the Study

Analytical Investigation of Prediction Error

The problem was attacked both analytically and empirically. Chapter 3 was concerned with the analytical discussion of prediction error and Chapters 4, 5, and 6 were associated with the empirical segment of the study. The analytical investigation initially assumed that the risk parameter, beta, was stable over time and that the probability distribution of the beta predictor was symmetric stable Paretian. It was found that if the beta predictors were independent, then prediction error, as measured by the scale parameter of the sampling distribution of the beta predictor, would vanish as the number of securities in the portfolio increased. If the independence assumption is relaxed then it is likely that prediction error persists at the portfolio level. However, because of a lack of information concerning dependent symmetric stable random variables, this statement is strictly valid for only one member of the symmetric stable family. For the normality case, it was shown that prediction error would approach the average covariance among the beta predictors.

Since recent empirical evidence indicates that the risk parameter was not stable over time and that the return distributions were more consistent with contaminated normal distributions, the problem of prediction error was also investigated analytically under these conditions. Assuming independence, it was found that as the number of securities increased in a portfolio, the prediction error approached the average cross product, $\overline{V_i V_j}$, of the nonstationary biases present on the individual betas. When the independence assumption was dropped, the average covariance among the betas was added to the average cross product term. In order to determine the impact of $\overline{V_i V_j}$ and the possible violation of the independence assumption on prediction error at the portfolio level, an empirical investigation was conducted (assuming the contaminated normal -- nonstationary beta case).

Empirical Investigation of Prediction Error

The methodology and design of the empirical study were described in Chapter 4. Basically, using a set of statistical tests of nonstationarity, two sets of securities were identified. One set required its securities to have stationary betas for two consecutive time periods.

This set was called the stationary approximation set. The other set required its securities to have stationary or nonstationary betas for the first time period and stationary betas for the subsequent time period. This set was called the nonstationary set.

The stationary approximation set had 90 securities which satisfied its requirements. Each of the two time periods for the stationary approximation set had 15 monthly observations. Within the set itself, 13 different portfolios were formed. The portfolios were either risk-partitioned or randomly partitioned.

The nonstationary set had 153 securities which satisfied its membership requirements. The first period of this set had 86 monthly observations and the second period had 14 monthly observations. Within the set, there were 17 different portfolios. These portfolios were also either risk-partitioned or randomly partitioned.

The next chapter, Chapter 5, described the beta predictors used in the empirical study and the prediction error evaluation criteria. The main predictor considered in the study was the OLS predictor. Using the observations in the first period, OLS was used to estimate the beta of the second period. A second predictor, consisting of a Bayesian adjustment of the OLS predictors was also identified. This predictor received limited application in the study.

Chapter 5 also presented and discussed four evaluation criteria that were used to determine if diversification could eliminate prediction error. The four criteria were the coefficient of variation, average percentage deviation, root prediction error, and the statistical tests of nonstationarity. The coefficient of variation is a relative measure of dispersion. The dispersion of the sampling distribution of the estimated portfolio beta is a measure of the prediction error on the true portfolio beta of the holding period (assuming the absence of nonstationary effects). If prediction error decreases as the number of securities in a portfolio increases, then so will the coefficient of variation.

The average percentage deviation was defined as the difference between the estimated betas of the prediction and holding periods, divided by the estimated holding period beta. It was shown that if prediction error decreases by diversification, then APD also decreases. Next, the measure, $\hat{RPE}$, was derived and defined. It was shown that $\hat{RPE}$ was a direct estimate of the prediction error on beta. And, finally, the statistical tests were used to detect the absence or presence of the nonstationary component of prediction error at the portfolio level.

The last content chapter, Chapter 6, contains an analysis of the results of the empirical study. The analysis had two major parts. First, the stationary approximation set of portfolios was examined. The evidence indicated that the major source of prediction error for this set was sampling error. The three criteria, APD, RPE, and CV_P all indicated that diversification was unable to eliminate prediction error for this set. Nonindependence of the beta predictors seemed to be a contributing cause.

The next part of the empirical analysis was concerned with the nonstationary set. The coefficient of variation indicated that sampling error was not a major component of prediction error for this set. This result was achieved by increasing the observations used in the prediction model. However, the statistical tests gave evidence of the presence of a nonstationary error component at the portfolio level. Moreover, both APD and $\hat{RPE}$ indicated that a nontrivial amount of prediction error still remained at the portfolio level (for both OLS and Bayesian predictors). Again the evidence was not in favor of diversification. Thus, it appears that risk information might be of value to the individual investor (i.e., to the extent that it allows further improvement in risk prediction and if the benefits of such improvement exceed the cost of using the information). Therefore, there exists a possible role for accounting information in portfolio analysis.

Suggestions for Future Research

The results of this study are necessarily dependent on the prediction models employed. It is conceivable that diversification would yield more favorable results if different prediction models were used. For example, any prediction model that could produce independent, unbiased beta predictors would result in the conclusion that diversification works. Efforts to discover such prediction models might be worthwhile.

There are several other areas of possible research. Since it does not appear that diversification can eliminate prediction error, additional research on how investors use or should use accounting variables to predict risk is needed. A related area is the identification of accounting variables which signal a change in the systematic risk — and in what manner such variables behave when the systematic risk does change. An interesting question is whether accounting variables have the ability to predict the rate and direction of change of beta. Also, in terms of portfolio analysis, it would seem desirable to classify individual securities according to the stability of their individual betas. Securities with the more stable betas would tend to be more desirable as the uncertainty surrounding them is less. Perhaps using accounting variables, statistical analysis, and other economic variables, securities could be placed in different stability classes.

APPENDIX A

LIST OF FIRMS

Firm No.	Firm Name	Compustat Industrial No.
1	Adams Millis Corporation	2300
2	Aguirre Company	6552
3	Alcan Aluminum Ltd	3334
4	Allied Supermarkets Incorporated	5411
5	Amax Incorporated	1000
6	American Distilling Company	2085
7	American Investment Company	6145
8	American Ship Building Company	3731
9	American Stores Company	5411
10	Ametek Corporation	3811
11	Amrep Corporation	6552
12	Amstar Corporation	2062
13	Anderson Clayton Company	2070
14	Archer Daniels Midland Company	2070
15	Arizona Public Service Company	4911
16	Armstrong Cork Company	2270
17	Armstrong Rubber Company	3000
18	Associated Dry Goods Corporation	5311
19	Atlantic Richfield Company	2912
20	Avco Corporation	9997

Firm No.	Firm Name	Compustat Industrial No.
21	Baker Oil Tools Incorporated	3533
22	Bausch Lomb Incorporated	3831
23	Bearings Incorporated	3714
24	Belding Heminway Incorporated	2200
25	Bell Howell Incorporated	3861
26	Bendix Corporation	3714
27	Beneficial Corporation	6145
28	Boeing Company	3721
29	Borden Incorporated	2020
30	Briggs Stratton Corporation	3560
31	Brunswick Corporation	3948
32	Burlington Industries Incorporated	2200
33	Callahan Mining Corporation	1021
34	Carpenter Technology Corporation	3311
35	Carter Hawley Hale Stores Incorporated	5311
36	Central Maine Power Company	4911
37	Chase Manhattan Corporation	6021
38	Chock Full O' Nuts Corporation	2099
39	Cincinnati Gas and Electric Company	4911
40	Cincinnati Milacron Incorporated	3540
41	City Investing Company	9997
42	Clark Oil Refining Corporation	2911
43	Coastal States Gas Corporation	4922
44	Coca Cola Bottling Company	2086

Firm No.	Firm Name	Compustat Industrial No.
45	Colgate Palmolive Company	2841
46	Commonwealth Edison Company	4912
47	Continental Copper and Steel Industries	3350
48	Copperweld Corporation	3311
49	Cowles Communications Incorporated	6710
50	Crompton Knowles Corporation	2803
51	Crouse Hinds Company	3610
52	Cummins Engine Incorporated	3713
53	D P F Incorporated	7394
54	Dan River Incorporated	2200
55	Deere Company	3522
56	Del Monte Corporation	2030
57	Deltec International Ltd	6199
58	Deltona Corporation	6552
59	Detroit Edison Company	4912
60	Disney Walt Productions	7949
61	Dr. Pepper Company	2086
62	Dover Corporation	3550
63	Dresser Industries Incorporated	3533
64	E G G Incorporated	3825
65	Eastman Kodak Company	3861
66	Edison Brothers Stores Incorporated	5661
67	Elgin National Industries Incorporated	1511
68	Emerson Electric Company	3610

Firm No.	Firm Name	Compustat Industrial No.
69	Emery Industries Incorporated	2803
70	Equitable Gas Company	4924
71	Esmark Incorporated	2010
72	Exxon Corporation	2913
73	Fairchild Industries Incorporated	3721
74	Financial Federation Incorporated	6125
75	Florida Power and Light Company	4912
76	Food Fair Stores Incorporated	5411
77	G F Business Equipment Incorporated	2520
78	General Cinema Corporation	7831
79	Gerber Products Company	2000
80	Giant Portland Cement Company	3241
81	Gibraltar Financial Corporation California	6125
82	Gould Incorporated	3714
83	Great Atlantic and Pacific Tea Incorporated	5411
84	Great Northern Nekoosa Corporation	2600
85	Gulton Industries Incorporated	3670
86	Harnischfeger Corporation	3536
87	Helene Curtis Industries Incorporated	2844
88	Hershey Foods Corporation	2065
89	Hewlett Packard Company	3825
90	Hoffman Electronics Corporation	3662
91	Holiday Inns Incorporated	7017
92	House Fabrics Incorporated	5949

Firm No.	Firm Name	Compustat Industrial No.
93	Household Finance Corporation	6145
94	Houston Light and Power Company	4912
95	Hydrometals Incorporated	3941
96	I N A Corporation	6332
97	I U International Corporation	9997
98	Idaho Power Company	4912
99	Ideal Toy Corporation	3941
100	Imperial Corporation of America	6125
101	Indianapolis Power and Light Company	4912
102	Ingersoll Rand Company	3560
103	International Nickel Company	1000
104	Interstate Brands Corporation	2051
105	Itek Corporation	3831
106	Johnson and Johnson	2837
107	Jonathan Logan Incorporated	2300
108	Kaiser Aluminum and Chemical Corporation	3334
109	Kansas Power and Light Company	4192
110	Kaufman Broad Incorporated	6500
111	Keystone Construction Industries Incorporated	3311
112	Laclede Gas Company	4924
113	Lane Bryant Incorporated	5600
114	Leaseway Transportation Corporation	4210
115	Lehigh Portland Cement Company	3499
116	Liberty Loan Corporation	6312

Firm No.	Firm Name	Compustat Industrial No.
117	Liggett Myers	2111
118	Londontown Corporation	2300
119	Lone Star Industries Incorporated	3241
120	Louisiana Land and Exploration Company	1311
121	Macy R H and Company Incorporated	5311
122	Mapco Incorporated	1311
123	Marcor Incorporated	5311
124	Marine Midland Bank	6023
125	Marquette Company	3241
126	McNeil Corporation	3550
127	Meredith Corporation	2721
128	Monsanto Company	2801
129	Moore McCormack Resources	4400
130	Morse Shoes Incorporated	5661
131	Morton-Norwich Products	2836
132	Mountain Fuel Supply Company	4922
133	N C R Corporation	3570
134	N L Industries Incorporated	3350
135	Narco Scientific Industries	3841
136	Nashua Corporation	3579
137	North American Coal Corporation	1211
138	Northern Indiana Public Service Company	4912
139	Oxford Industries Incorporated	2300
140	Pacific Power and Light Company	4911

Firm No.	Firm Name	Compustat Industrial No.
141	Panhandle Eastern Pipe Line Company	4922
142	Pasco Incorporated	6710
143	Peoples Gas Company	4924
144	Phelps Dodge Corporation	3331
145	Philadelphia Electric Company	4912
146	Potomac Electric and Power Company	4911
147	Products Research and Chemical Corporation	2899
148	Pueblo International Incorporated	5411
149	Pullman Incorporated	3740
150	Quaker Oats Company	2000
151	Rapid-American Corporation	5331
152	Revco D S Incorporated	5912
153	Rochester Gas and Electric Corporation	4911
154	Rohr Industries Incorporated	3725
155	Rollins Incorporated	7399
156	Royal Crown Cola Company	2086
157	Savannah Electric and Power Company	4912
158	Servomation Corporation	5962
159	Sherwin Williams Company	2850
160	Singer Company	3630
161	Skaggs Companies	5912
162	South Jersey Industries	4924
163	Southern Pacific Company	4011
164	Sprague Electric Company	3679

Firm No.	Firm Name	Compustat Industrial No.
165	Springs Mills Incorporated	2200
166	Stone Webster Incorporated	7399
167	Stone Container Corporation	2650
168	Sucrest Corporation	2062
169	Systron Donner Corporation	3825
170	Talley Industries	3871
171	Tampa Electric Company	4912
172	Technicolor Incorporated	7810
173	Tektronix Incorporated	3825
174	Texas Industries	3679
175	Thomas Industries	3642
176	Trans World Airlines Incorporated	4511
177	Triangle Industries	3350
178	Twentieth Century Fox Film Corporation	7810
179	Tyco Laboratories Incorporated	3679
180	U A L Incorporated	4511
181	U S M Corporation	3550
182	Unilever N V	2841
183	Union Corporation	3499
184	Union Electric Company	4911
185	United Brands Company	2010
186	Upjohn Company	2835
187	Varian Associates	3825
188	Wallace Murray Corporation	3430

Firm No.	Firm Name	Compustat Industrial No.
189	Warnaco Incorporated	2300
190	Warner Communications Incorporated	3652
191	Wean United Incorporated	3540
192	Western Bancorporation	6027
193	Westinghouse Electric Corporation	3600
194	Weyerhaeuser Company	2400
195	Wickes Corporation	5211
196	Zale Corporation	5944
197	Zapata Corporation	1511

APPENDIX B

NONSTATIONARY TESTS

The description of the TIMVAR tests is based on the paper written by Brown et al. (1973). The basic regression model considered in this study is:

$$Y_t = \alpha_t + \beta_t X_t + U_t \quad t = 1, \ldots, T,$$

where Y_t is the observation of the dependent variable at time t and X_t is the observation of the independent variable at time t. The error terms, U_t, are assumed to be independent and normally distributed with means of zero and variances, Var_t, t = 1, . . ., T. The hypothesis of constancy over time is investigated by four tests -- cusum test, cusum of squares test, homogeneity tests of moving regressions and Quandt's log-likelihood ratio test. This hypothesis is formally expressed as:

$$H_o: \begin{pmatrix} \alpha_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \beta_2 \end{pmatrix} = \ldots \begin{pmatrix} \alpha_T \\ \beta_T \end{pmatrix} .$$

Cusum Test

The cusum test is based upon the use of recursive residuals. Recursive residuals are standardized residuals which are calculated by inserting X_t in the regression equation calculated from the first t - 1 observations, and then subtracting this predicted value from the actual value, Y_t. That is, assuming H_o to be true, let $\hat{\alpha}_{r-1}$ and $\hat{\beta}_{r-1}$ be the OLS estimates obtained from the first r-1 observations. The forward recursive residual is thus defined as:

$$w_r = [Y_r - (\hat{\alpha}_{r-1} + \hat{\beta}_{r-1}X_r)] \;/\; [1 + 1/r + (X_r - \bar{X})^2 \;/\; \Sigma(X_i-\bar{X})^2]^{1/2}$$

$r = 3, \ldots, T$.

Under H_o it can be shown that $w_{k+1}, \ldots, w_T$ are independent, N(0,Var) (where k = number of regressors). If the coefficients alpha and beta are constant up to a certain point and then change, the w_r's will have zero means up to the disturbance point and non-zero thereafter. The cusum test examines plots of the cusum quantity,

$$W_r = (1/S) \sum_{k+1}^{r} w_j, \; r = 3, \ldots, T$$

against r for r = k+1, . . ., T, where S is the estimate of the standard deviation of the residuals using all T observations.

Since the w_r's are N(0,Var) the W_r's are approximately normal such that:

$$E(W_r) = 0, \; Var(W_r) = r-2, \text{ and } Covar\,(W_r,W_s) = \min(r,s) - 2.$$

Next, a pair of symmetrical lines, above and below, the mean value line $E(W_r) = 0$ are constructed such that the probability of the sample path crossing one of the lines is alpha, the level of significance. The method of construction is based upon known results in Brownian motion theory. A backward recursion is also performed to assist in locating the disturbance point. Critical values for alpha = .01, .05, and .1 are, respectively, 1.143, .948, and .85 (these critical values apply, of course, to the cusum test). If a computed W_r is greater than the critical value selected, the null hypothesis is rejected.

Cusum of Squares Test

The cusum of squares test is based upon the squared recursive residuals and examines the plot of the quantities:

$$s_r = \left(\sum_{k+1}^{r} w_j^2 \right) / \left(\sum_{k+1}^{T} w_j^2 \right) \qquad r = 3, \ldots, T$$

The test is more sensitive to haphazard changes in coefficients than is the cusum test. It is also sensitive to changes in the residual variance. Under H_o, s_r can be shown to have a beta distribution with mean $(r-2)/(T-2)$. A pair of lines, $(r-2)/(T-2) \pm c_o$[1] are drawn parallel to the mean value line such that the probability that the sample path of s_r crosses a line is alpha.

Homogeneity and Chow Tests

Another means for investigating beta instability is the use of moving regressions. A regression is fit on a short segment of n observations which is then moved along the series. That is, for T observations, regressions are fit on the segments, (1,n), (2,n+1), . . . (T - n - 1, T). Graphs of the coefficients of the segments provide visual evidence of departures from constancy. A significance test for constancy of regression coefficients, the homogeneity test, was also utilized. It is based on the use of regressions on nonoverlapping time segments using analysis of variance. The nonoverlapping time segments for a moving regression of length n, are (1,n), (n+1, 2n), . . ., [(p-1)n + 1, T], where p is the integral part of T/N. The homogeneity statistic for b nonoverlapping segments is:

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1. For the derivation of c_o, see Brown et al. (1973, pp. 154-155).

$$F = [(T-2p)/(2p-2)][(SSE_T - SSE_1 - SSE_2 - \ldots - SSE_b)/(\sum_1^b SSE_i)]$$

where SSE_T is the residual sum of squares for the regression on the entire T observations and SSE_i, i = 1, . . ., b is the residual sum of squares for the regression on each of the nonoverlapping segments. If b = 2, then the above F test is equivalent to the Chow test.

Quandt's Log-likelihood Test

This test is used to detect the point in time in which the regression relationship changed from one constant relationship to another constant relationship. Quandt (1958) describes the development of the techniques. For each r from r = 3 to T - 3 the ratio $Q_r = \log_{10}$ [(max likelihood of observations given H_o) / (max likelihood of observations given H_1)] is computed, where H_1 is the hypothesis that observations in the time segments (1, . . ., r) and (r+1, . . ., T) come from two different regressions. The minimum value of Q_r is the estimate of the point at which the switch from one relationship to another has occurred. It can be shown that $Q_r = (r\log S_1^2)/2 + ((T-r)/2)\log S_2^2 - (T\log S^2)/2$, where S_1^2, S_2^2, and S^2 are the residual sum of squares divided by the number of observations in each of the subintervals and the entire interval, respectively.

APPENDIX C

SELECTION PROCESS EXAMPLE

Tables C.1, C.2, and C.3 contain the results of three computer runs that illustrate the selection process for six securities. The results associated with these six securities are typical of the larger process described in Chapter 4. Table C.1 has the results of the first computer run for the six securities. The results of the second run are found in Table C.2 and those of the third in Table C.3.

In the first computer run, only firm 5 displayed behavior consistent with stability for the entire 100 observation interval. This security qualifies for membership in either set of portfolios and receives no further consideration. However, the test results in Table C.1 indicate that the other five securities had at least one Timvar test showing significance at the .05 level. These five securities qualified for a second computer run.

The interval for each of the five securities was divided at the point indicated by Quandt's log-likelihood ratio test. For example, the point of division for firm 74 was at the 80th observation. The Timvar tests were run on observations 1-80 and 81-100, respectively. Thus, a total of ten intervals, two for each firm, received application of the Timvar tests. Of the five firms, two were classified as having unstable betas in the 100 observation interval, one as indeterminate, and two qualified for further investigation in the third computer run.

Table C.1. The First Computer Run.

	Firm					
	5	53	74	6	176	162
β	.739	1.91	1.78	1.05	2.25	.657
Cusum BW	.703	.618	.436	.867	.745	.359
Cusum Sq.BW	.121	.145	.159	.419[a]	.127	.246[a]
Cusum FW	.631	.835	.543	.311	.525	.382
Cusum Sq.FW	.136	.163	.138	.423[a]	.104	.247[a]
MR1	1.12	2.46 [a]	1.68	.977	1.57	.223
MR2	1.33	2.63 [a]	2.18 [a]	1.42	2.19 [a]	.604
Quandt	3	52	80	69	56	79

a. Significant at a level of .05.

Table C.2. The Second Computer Run.

	Firm 53		Firm 74		Firm 6		Firm 176		Firm 162	
	1	2	1	2	1	2	1	2	1	2
β	3.37	0.886	2.52	0.875	0.744	1.28	1.48	2.80	0.503	0.811
Cusum BW	0.394	0.611	0.265	0.324	0.787	0.518	0.996[a]	0.948[a]	0.402	0.422
Cusum Sq.BW	0.180	0.116	0.234[a]	0.136	0.210	0.297	0.210[a]	0.170	0.210[a]	0.291
Cusum FW	0.806	0.451	0.615	0.721	0.241	0.583	0.460	0.489	0.501	0.212
Cusum Sq.FW	0.144	0.115	0.244[a]	0.092	0.184	0.292	0.204	0.228	0.22 [a]	0.251
MR1	0.981	1.15	1.62	0.150	0.996	0.870	1.31	1.57	0.374	0.147
MR2	1.54	1.42	0.950	2.29	0.583	0.810	1.24	0.554	0.480	0.990
Quandt	--	--	--	--	--	--	22	30	50	--
F	1.33		1.00		6.57[a]		1.6[a]		3.14[a]	
Chow	9.50[a]		6.38[a]		1.67		4.6[a]		1.60	

a. Significant at a level of .05.

Table C.3. The Third Computer Run.

	Firm 176				Firm 162	
	1	2	3	4	1	2
β	0.431	1.84	3.85	2.44	0.553	0.334
Cusum BW	0.303	0.819	0.568	0.789	0.739	0.328
Cusum Sq.BW	0.110	0.154	0.160	0.291	0.297[a]	0.341[a]
Cusum FW	0.414	0.604	0.623	0.468	0.497	0.496
Cusum Sq.FW	0.139	0.117	0.112	0.331	0.269[a]	0.289[a]
MR1	1.48	0.847	1.02	0.60	0.259	0.406
MR2	1.92	1.56	0.195	2.20	0.271	0.492
Quandt	--	--	--	--	42	16
F	2.92[a]		2.11		3.42[a]	
Chow	1.30		3.32[a]		.738	

a. Significant at a level of .05.

Firms 53 and 74 had betas classified as unstable. As Table C.2 shows, Firm 53 had results consistent with stability in each of the subintervals and also had F tests results indicating a stable variance. The Chow test for Firm 53 was also significant indicating that the betas of the two subintervals were significantly different. Firm 74 had significant results for the cusum of squares test (forward and backward) in the first interval and had results consistent with stability in the second interval. The Chow test indicated a significant difference between the betas of the two subintervals. For purposes of forming the nonstationary set of portfolios, this result was considered as adequate identification of a nonstationarity. Firm 6 had subintervals consistent with stability but had an F test indicating nonconstant residual variance. Also, the Chow test did not show a significant difference. This firm was classified as indeterminate. Finally, Firms 176 and 162 had at least one subinterval with unstable indications and qualified for further investigation on the third computer run.

Firm 176 had two subintervals that were each subdivided and Firm 162 had one (i.e., the first) that was subdivided (again, the same criterion was used to subdivide the intervals). According to Table C.3, the four subintervals of Firm 176 all have stable indications. The Chow test indicated that the betas of the third and fourth subintervals were significantly different. Thus, a nonstationarity was identified. Firm 162 still has unstable indications and the variance is not constant across subintervals. This firm (and others like it) was classified as indeterminate (a fourth run was not made on firms such as these since by this time it was felt that a sufficient number of nonstationarities had been identified to permit the formation of the desired portfolios).

APPENDIX D

PORTFOLIO COMPOSITION

The securities constituting the various portfolios are listed below according to firm number. The firm number corresponds to the same number in Appendix A. The firm's name and industry can be found by referring to Appendix A. The securities of the nonstationary set which were identified as having a nonstationarity within the prediction period interval are marked with an asterisk superscript. The portfolios are identified by the same symbols used in Chapter 4 of this study.

Stationary Approximation Set

Hi
1, 8, 10, 13, 22, 23, 26, 38, 47, 50, 53, 60, 64, 73, 78, 79, 91, 108, 119, 127, 135, 144, 151, 155, 156, 165, 172, 178, 183, 187

M
2, 6, 7, 40, 44, 51, 54, 90, 97, 110, 121, 180, 186, 190, 193, 194, 24, 27, 28, 29, 52, 57, 134, 142, 152, 160, 171, 176, 177, 191

L
61, 65, 68, 75, 77, 115, 116, 128, 131, 140, 161, 163, 168, 189, 195, 4, 5, 17, 34, 42, 66, 67, 71, 107, 109, 113, 118, 139, 159, 181

R1

23, 24, 27, 44, 47, 66, 90, 113, 119, 131, 134, 135, 139, 144, 159, 160, 161, 163, 165, 171, 172, 174, 176, 177, 180, 181, 183, 186, 187, 189

R2

190, 191, 194, 195, 2, 6, 8, 17, 28, 34, 51, 53, 57, 60, 61, 65, 73, 78, 108, 115, 126, 128, 151, 152, 155, 156, 168, 178, 1

R3

5, 7, 10, 13, 22, 26, 29, 38, 40, 42, 50, 52, 54, 64, 67, 71, 75, 77, 79, 91, 96, 97, 107, 109, 110, 116, 118, 121, 140, 142

Hi-1 (the securities added to Hi)

2, 6, 7, 40, 44, 51, 54, 90, 97, 110, 121, 180, 186, 190, 193

Hi-2 (the securities added to Hi-1)

194, 24, 27, 28, 29, 52, 57, 134, 142, 152, 160, 171, 176, 177, 191

Hi-3 (the securities added to Hi-2)

61, 65, 68, 75, 77, 115, 116, 128, 131, 140, 161, 163, 168, 189, 195

Hi-4 (the securities added to Hi-3)

4, 5, 17, 34, 42, 66, 67, 71, 107, 109, 113, 118, 139, 159, 181

R1-1 (the securities added to R1)

2, 8, 28, 51, 53, 108, 127, 128, 155, 168, 190, 191, 193, 184, 195

R1-2 (the securities added to R1-1)

1, 6, 17, 34, 61, 65, 73, 78, 101, 115, 129, 151, 152, 156, 178

R1-3 (the securities added to R1-2)

5, 10, 13, 22, 29, 38, 50, 52, 54, 64, 67, 71, 75, 96, 140

R1-4 (the securities added to R1-3)

26, 40, 42, 77, 79, 91, 97, 107, 109, 110, 116, 118, 121, 142, 7

Nonstationary Set

Hi

1, 2*, 7, 11*, 20, 21*, 24*, 25, 27, 31*, 38, 41, 47*, 49, 53*, 60*, 64*, 69, 73*, 74*, 78*, 82*, 85, 87*, 89, 90*, 91, 92, 100*, 105, 108, 114, 119, 121*, 125, 129*, 135, 142*, 147, 151, 164, 169*, 170*, 172, 173*, 176*, 178*, 183, 187*, 191

M

4, 8, 13*, 16, 17, 18, 19*, 23, 28, 32, 34, 37, 40, 50, 52, 54, 55*, 61*, 62, 63*, 67, 68*, 71, 77, 80, 84, 93*, 96, 97, 113*, 115*, 116, 122, 127, 128*, 131, 134*, 137, 139, 144, 152, 160, 161*, 163*, 174*, 175, 177, 181, 186, 188, 189

L
3*, 5, 6, 9, 12, 14, 15, 29, 30*, 33, 36, 39, 45, 46, 56, 59, 65*, 70, 72*, 75, 76, 79, 83, 88, 94, 103, 106*, 109, 111, 112, 117*, 126, 132*, 138, 140, 141*, 143, 145*, 146, 148*, 150, 153, 154, 157, 162, 165, 167, 168, 171, 182, 184

R1
7, 16, 18, 19*, 21*, 36, 37, 38, 59, 65*, 68*, 75, 77, 94, 100*, 105, 111, 114, 116, 119, 121*, 125, 126, 127, 134*, 143, 144, 145*, 146, 147, 148*, 150, 153, 154, 157, 160, 161*, 162, 163*, 165, 167, 168*, 169, 171, 172, 173*, 174*, 175, 176*, 178*, 181

R2
2*, 4, 5, 6, 9, 11*, 13*, 14, 15, 23, 25, 28, 32, 40, 45, 47*, 49, 50, 52, 53*, 54, 56, 63*, 67, 71, 73*, 76, 80, 85, 89, 92, 93*, 96, 106*, 109, 118*, 112, 113*, 115*, 122, 132*, 135, 137, 138, 139, 140, 141*, 151, 152, 164, 177

R3
1, 3*, 8, 12, 17, 20, 24*, 27, 29, 30*, 31*, 33, 34, 39, 41, 46, 55*, 60*, 61*, 62, 64*, 69, 70, 72*, 74*, 78*, 79, 82*, 83, 84, 87*, 88, 90*, 91, 97, 103, 108, 117*, 128*, 129*, 131, 142*, 170*, 182, 183, 184, 186, 187*, 188, 189, 191

Hi-1 (securities added to Hi)

4, 8, 18, 19*, 28, 54, 55*, 67, 96, 97, 116, 127, 134*, 144, 152, 175, 188

Hi-2 (securities added to Hi-1)

16, 17, 34, 50, 52, 62, 63*, 68*, 71, 80, 115, 122, 128*, 131, 139, 174*, 181

Hi-3 (securities added to Hi-2)

13*, 23, 32, 37, 40, 61*, 77, 84, 93*, 113*, 137, 160, 161*, 163*, 177, 186, 189

Hi-4 (securities added to Hi-3)

14, 15, 29, 33, 56, 65*, 75, 76, 94, 103, 132*, 141*, 148*, 150, 154, 168, 171

Hi-5 (securities added to Hi-4)

3*, 5, 6, 30*, 39, 45, 46, 79, 106*, 111, 126, 138, 140, 145*, 157, 165, 167

Hi-6 (securities added to Hi-5)

9, 12, 36, 59, 70, 72*, 83, 88, 109, 112, 117*, 143, 146, 153, 162, 182, 184

R1-1 (securities added to R1)

13*, 40, 73*, 80, 89, 92, 93*, 110*, 113*, 132*, 138, 139, 140, 141*, 151, 177

R1-2 (securities added to R1-1)

2*, 15, 23, 25, 32, 45, 47*, 50, 53*, 54, 63*, 71, 76, 96, 109, 135, 137

R1-3 (securities added to R1-2)

4, 5, 6, 9, 11*, 14, 28, 49, 52, 56, 67, 85, 106*, 115*, 122, 152, 164

R1-4 (securities added to R1-3)

8, 12, 17, 20, 24*, 27, 30*, 39, 55*, 64*, 69, 70, 72*, 74*, 87*, 90*, 170*

R1-5 (securities added to R1-4)

3*, 29, 34, 60*, 78*, 82*, 83, 84, 88, 91, 97, 103, 117*, 129*, 131, 142*, 184

R1-6 (securities added to R1-5)

1, 31*, 33, 41, 46, 61*, 62, 79, 108, 128*, 182, 183, 186, 187*, 188, 189, 191

APPENDIX E

RPE EXAMPLE

The discrete population is listed in Table E.1. The values listed are actual returns for a portfolio, R_{pH}, and for the market, R_{mH}, for a period of ten months. The values are assumed to be equally likely. The regression equation calculated from the data in Table E.1 is:

$$R_{pH} = .00265 + 1.093\ R_{mH} + e_{pH} \quad .$$

The variance of the residual, e_{pH}, is equal to .005072. Assume the predictors are $\hat{\alpha} = .00665$ and $\hat{\beta} = 1.493$. The actual prediction error is $U_{\alpha} = .004$ and $U_{\beta} = .4$.

Calculation of the Population RPE

Using the data from Table E.1, RPE can be calculated as follows:

$$(RPE)^2 = [E(z_p^2 \mid \hat{\alpha}, \hat{\beta}) - Var(e_{pH})] \,/\, E(R_{mH}^2) \qquad (E.1)$$

$$= (.0066287 - .005072) \,/\, .00977$$

$$= .1593$$

and

$$RPE = .3992 \; .$$

There is an error of .0008 in the estimation of U_{β}. The dropped terms $U_{\alpha}^2 + 2U_{\alpha}U_{\beta}E(R_{mH})$ equal -.0000064, which indicates the validity of the assumption that these terms have a negligible effect on the estimation of prediction error for beta.

Table E.1. Discrete Population Values.

Month	R_{pH}	R_{mH}	z_p^2
1	.01	.03	.001717
2	-.16	-.09	.001042
3	.03	-.03	.0046431
4	-.14	-.12	.0010569
5	-.10	-.14	.0104796
6	.11	.17	.022638
7	-.05	-.05	.000324
8	-.13	-.04	.005918
9	.25	.08	.015354
10	.13	.12	.003115

In practice, the actual population values used in the right side of equation (E.1) will not be available. Rather, unbiased sample estimates of the values must be used. For purposes of illustration, a sample of 7 was taken from the population of Table E.1. The sample data are listed in Table E.2. The sample estimate of the residual variance was .0083.

Calculation of the Sample Estimate of RPE

Using the data from Table E.2, the sample estimate of RPE can now be calculated:

$$(\hat{RPE})^2 = [\hat{E}(z^2 \mid \hat{\alpha}, \hat{\beta}) - \hat{Var}(e_{pH})] / \hat{E}(R^2_{mH}) \quad \text{(E.2)}$$

$$= (.00976 - .0083) / .01687$$

$$\hat{RPE} = .294$$

where

$$\hat{E}(z^2 \mid \hat{\alpha}, \hat{\beta}) = [(N-1)/N]S^2_z + (\bar{z})^2$$

$$= (6/7)(.011354) + .000023$$

$$= .00976,$$

and

$$\hat{E}(R^2_{mH}) = [(N-1)/N]S^2_{R_{mH}} + (\bar{R}_{mH})^2$$

$$= (6/7)(.01944) + .0002$$

$$= .01687.$$

It is expected that the estimate, $\hat{RPE}$, will improve as the estimates on the right hand side of (E.2) improve. The quality of these estimates is a function of the number of observations in the sample. For this study the holding period values were used in calculating the

Table E.2. Sample Values.

R_{pH}	R_{mH}	z_p^2	z_p
.01	.03	.001717	-.04129
-.16	-.09	.001042	-.03228
.03	-.03	.004643	-.06814
-.14	-.12	.001057	.03251
-.10	-.14	.010480	.10237
.11	.17	.022638	-.15046
.25	.08	.015354	.12391

estimates. The holding periods of the stationary approximation set and the nonstationary set had, respectively, 15 observations and 14 observations.

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